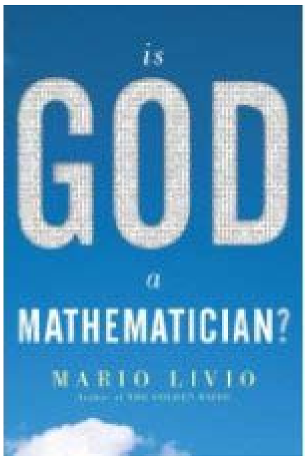
'Is God a mathematician?'

reviewed by Marianne Freiberger



Is God a mathematician?

by Mario Livio

"Oh god, I hope not," was the reaction of a student when Livio asked the title question at a lecture, and it's a reaction that's likely to be replicated by many unsuspecting bookshop browsers. But despite its frightening title, the book's appeal could not be broader. All that's required to enjoy this fascinating read is a capacity to marvel at the mysteries of nature and humankind's attempts to understand them. This is "popular mathematics", or rather "popular philosophy", in the true sense of the word.

The question of whether god is a mathematician refers to the apparently omnipotent powers of mathematics to describe the world we live in — its "unreasonable effectiveness", to use a phrase coined by physics Nobel Laureate Eugene Wigner in 1960. Evidence of this omnipotence is everywhere. The laws of physics, the movements of the stock market (though it may be hard to believe right now), the way our brain works, even chance events: all can be described in the language of mathematics. What's more, the mathematics required to solve a particular problem, for example to describe the nature of sub-atomic particles, has often been developed decades, or even centuries, before the problem was first posed. Time and time again, mathematics just happens to fit the bill perfectly. So is mathematics woven into the fabric of nature, independent of the human mind, but there for us to discover? Or is it a human invention? If it's the latter, then why does it apply to external physical phenomena at all?

These questions, as Livio points out, are not new. It was of course an ancient Greek — Plato — who postulated that our experienced reality is a mere shadow of an absolute world of ideas, in which ideal mathematical objects — lines, circles, and so on — find their rightful place. Platonist views did indeed dominate mathematical thinking for much of the long history of maths. From ancient Greece onwards (and Plato is by no means the only culprit here), many mathematicians regarded the objects of mathematics as existing independently of the human mind and spelling out an eternal truth of their own. Mathematics, then, is a matter of discovery, rather than invention. Nothing could be nobler than to pursue this truth for its own sake, and many mathematicians, including the ancient giants Pythagoras, Plato, and Archimedes, as well as modern-day mathematicians including G.H. Hardy, held a healthy disdain for those who used mathematics as a mere tool for practical problem solving, or, god forbid, profit making. Plato, as Livio tells us, thought that even astronomers should "leave the heavens alone" and concentrate on the "laws of motion in some ideal, mathematical world, for which the observable heaven is a mere illustration."

Platonism has powerful proponents even today. Livio points to the renowned mathematician Roger Penrose, who extends the mystery beyond the two-pronged vision of a world of Platonic ideas versus a world of external reality by adding a third dimension: the human mind and its conscious perception of the world. The human mind has access to the world of mathematical ideas and emerges from the physical world, which in turn seems to obey the laws of mathematics. Why is it that mathematics seems to hold this mysterious triangle together?

Mathematical Platonism certainly feels right: even dinosaurs, or aliens, could not argue with the apparent universality of the natural numbers or Euclid's ideal geometric objects. But are we being misled by a sort of observational bias? Livio quotes the famous British mathematician Michael Atiyah, who said that if we were jellyfish living in the depths of an ocean, where there are no individual objects to count, we would have built our mathematics around continuous quantities, rather than the natural numbers that appear so universal to us. Viewed in this light, mathematics really is a mere human invention, fashioned to suit our perceptions.

To throw light on his mystery, Livio takes us on a whirlwind tour of over two millennia of mathematical history, alighting at periods that were populated by the greatest mathematical innovators, including, for example, Galileo, Descartes, and Newton. The bulk of the book, framed by an introductory chapter setting out the questions and a couple of concluding chapters probing for the answers, is devoted to this history. Livio's accounts of the great lives and work are guided by the questions he's trying to answer, but they are far from dry. Historical anecdotes and interesting facts liven up the pace and we gain plenty of insight into characters, as well as theory. Archimedes, so we learn, tended to forget food and personal hygiene when working on his geometry (some things never change) and Galileo was actually unable to withstand the pressure of the church and ended up issuing an unqualified retraction of his revolutionary views on the solar system. Even if the central questions of this book leave you cold, it's worth reading simply as an introductory, if selective, mathematical history.

Throughout the book, Livio does not assume any prior knowledge of mathematics, or philosophy and history for that matter. All concepts are explained, even simple ones like prime numbers, and there's hardly an equation to be found. Even for mathematical novices, this will be a comfortable slouch-on-the-sofa read, rather than a paper-and-pencil one. A chapter on logic does require some more serious thought than the rest, but this is no surprise given the level of abstraction of the ideas Livio is trying to bring across. Those who are reasonably familiar with the history of maths may not find much that's new, but I for one still enjoyed seeing familiar episodes put into the context of the central question.

Livio's historical accounts trace the changes in the understanding of mathematics through the ages, from the mathematical mysticism of the Pythagoreans, to Newton's superhuman effort to not only explain, but also predict physical reality, and with an incredible degree of accuracy at that. As a language of the Universe that resides in a world external to the human mind, maths did indeed rule supreme for a long time, until the nineteenth century when it was hit by a major crisis.

The root of the crisis dates back to Euclid's *Elements*. This work on geometry is probably the most-read book of all time apart from the bible — another illustration of the surprising longevity of mathematics. In the *Elements* Euclid set out the rules of mathematical exposition as they are still followed today: start with a set of axioms, self-evident truths that no-one in their right mind would call into doubt, and then proceed by deriving mathematical results from these axioms using only the rules of logic. Even Newton, when writing down his theory of gravitation which, after all, describes the real world rather than one of ideas, followed this strict axiomatic approach.

One of Euclid's original axioms, however, did not appear that self-evident, and for centuries mathematicians had tried to prove that it followed directly from the others, so that it could in fact be left out. In the nineteenth century a crisis ensued because it became clear not only that the axiom was independent of the others, but also that without it, you could build other geometries that were also perfectly free of contradiction. Euclid's geometry, which had been assumed to be universally woven into the fabric of nature, was only one of several self-consistent alternatives.

This devastating realisation, as Livio explains, caused a shift towards a view of mathematics as a game whose rules could be changed arbitrarily to create another game. The rules that western mathematicians throughout the centuries had chosen to adopt are by no means the only ones available, they are simply the ones that chime best with our perception of the abstract aspects of the physical world. Mathematics, then, should be viewed not only as a human product which is separate from the physical world, but also as being devoid of content: a collection of abstract objects whose interactions are governed by whatever set of rules you choose. The meat of mathematics were not Euclid's geometric shapes or Pythagoras's numbers, but the rules imposed by logic.

At the beginning of the twentieth century efforts were made to unify all of mathematics into a single abstract and axiomatic system. But a second crisis was soon to follow, as logicians proved that any axiomatic system which generates even only the simplest of mathematical objects, the natural numbers and their arithmetic, will always contain statements that no-one can ever prove to be true or false. In other words, there are things you can say of the natural numbers that are undoubtedly true, but which you'll never be able to derive from the axioms. But proving things from first principles is of course what mathematics is all about. This view of mathematics as not only a human thought experiment, but an "imperfect" one at that, is a long way from the divine truth envisioned by the likes of Plato and Descartes.

So then, is god a mathematician? After exploring what evolutionary psychologists have to say on the matter, Livio ventures some answers in his final chapter. The book is too much of a philosophical detective story for me to give the game away and tell you what they are. I'll restrict myself to an interesting point made by Livio regarding the question of whether mathematics is invented or discovered. He says it's both: mathematicians *invent* mathematical concepts, but they *discover* mathematical truths about them, because here they are guided by the rules of the game. Much like the umbrella was invented in England and not the Sahara, so was the concept of the golden ratio invented by the Greeks, and not the Indians or Chinese. The Greeks' preoccupation with geometry brought them into frequent contact with this ratio, and so they needed a name for it — beyond that, there's nothing universal about this particular object. Livio's point explains a remark made by Philip David and Reuben Hersh, that mathematicians are Platonists on weekdays, and non-Platonists on Sundays. When you're doing maths, you certainly feel like you're discovering, because you're constrained by the rules of the game — that's your weekday activity. But then when it comes to actually justifying your feeling that the things you've been working with all week are universal, it's easier to chicken out and pretend that you don't believe in their universality at all.

I won't give away more of Livio's own answers here. Whether or not you end up going along with them probably depends on how strong your prejudices about maths were to start with. I for one remain a convinced Platonist. But, as Livio points out, availing himself of the words of Bertrand Russell, it's not really the conclusion that counts, but the fun to be had while getting there.

To get a taste of Livio's book, read his article Unreasonable effectiveness in this issue of Plus.

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Unreasonable effectiveness

by Mario Livio

Mario Livio's book, Is God a mathematician is reviewed in this issue of Plus.

In 1960, physics Nobel Laureate Eugene Wigner wrote a famous article entitled *The Unreasonable Effectiveness of Mathematics in the Natural Sciences*. In this article, Wigner referred to the uncanny ability of mathematics not only to *describe*, but even to *predict* phenomena in the physical world. He wrote:

> *"The miracle of the appropriateness of the language of mathematics to the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve".*

Unreasonable effectiveness: how come mathematics describes nature's mysteries so well? This image shows the Orion Nebula as seen by the Hubble Space Telescope. Image courtesy NASA, ESA, M. Robberto (Space Telescope Science Institute/ESA) and the Hubble Space Telescope Orion Treasury Project Team.

Indeed, how is it possible that all the phenomena observed in classical electricity and magnetism can be explained by means of just four mathematical equations? Furthermore, physicist James Clerk Maxwell (after whom those four equations of electromagnetism are named) showed in 1864 that the equations predicted that varying electric or magnetic fields should generate propagating waves. These waves—the familiar electromagnetic waves—were eventually detected by the German physicist Heinrich Hertz in a series of experiments conducted in the late 1880s. And if that is not enough, the modern theory of electrodynamics, known as *quantum electrodynamics* (QED), is even more astonishing. In 2006 a group of physicists at Harvard University determined the magnetic moment of the electron (which measures how strongly the electron interacts with a magnetic field) to a precision of eight parts in a trillion. Calculations of the electron's magnetic moment based on QED reach the same precision and the two results agree! What is it that gives mathematics such incredible powers? In the present short article I will not even attempt to answer this intricate question. I will rather present some less familiar aspects of the problem itself.

The puzzle of the power of mathematics is in fact even more complex than the above example of electromagnetism might suggest. There are actually two facets to the "unreasonable effectiveness," one that I will call *active* and another that I dub *passive*. The *active* facet refers to the fact that when scientists attempt to light their way through the labyrinth of natural phenomena, they use mathematics as their torch. In other words, at least some of the laws of nature are formulated in directly applicable mathematical terms. The mathematical entities, relations, and equations used in those laws were developed for a specific application. Newton, for instance, formulated the branch of mathematics known as *calculus* because he needed this tool for his equations of motion. Similarly, string theorists today often develop the mathematical machinery they need. The *passive* effectiveness, on the other hand, refers to cases in which abstract mathematical theories had been developed with absolutely no applications in mind, only to turn out decades, or sometimes centuries later, to be powerfully predictive physical models. In what follows I will describe a wonderful example of the continuous interplay between active and passive effectiveness.

To be or knot to be

In the Greek legend of the Gordian knot Alexander the Great used his sword to slice through a knot that had defied all previous attempts to untie it. Knots, and especially maritime knots, enjoy a long history of legends and fanciful names such as "Englishman's tie," "hangman's knot," and "cat's paw". Knots became the subject of serious scientific investigation when in the 1860s the English physicist William Thomson (better known today as Lord Kelvin) proposed that atoms were in fact knotted tubes of ether (that mysterious substance that was supposed to permeate space). In order to be able to develop something like a periodic table of the elements, Thomson had to be able to classify knots—find out which different knots are possible. This particular need sparked a great interest in the mathematical theory of knots. This is a perfect example of what I dubbed the *active* aspect of the effectiveness of mathematics. In other words, physicists and mathematicians thought that knots were viable models for atoms, and consequently they enthusiastically engaged in the mathematical study of knots.

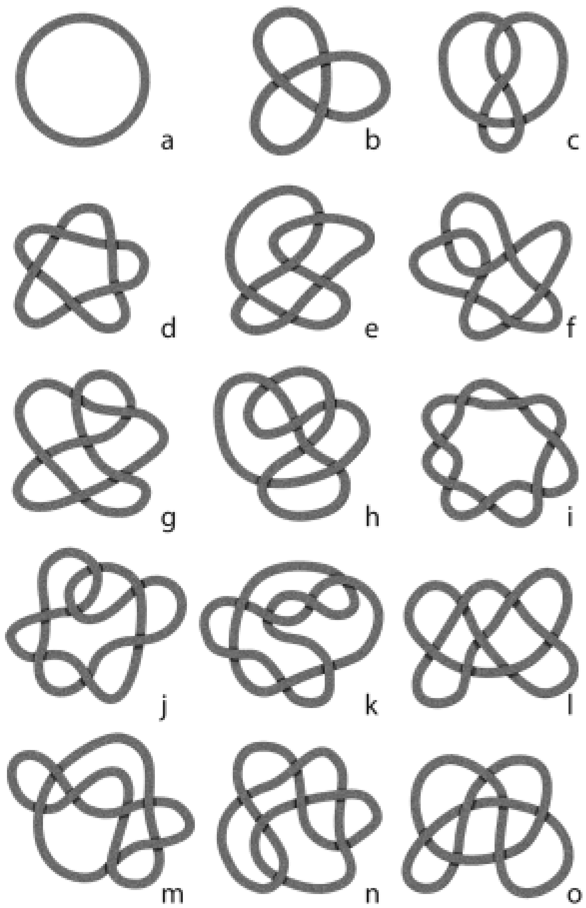

Figure 1: A selection of knots. Image created by Ann Feild.

A mathematical knot looks very much like a familiar knot in a string, only with the string's ends spliced (a few examples of knots are shown in figure 1). In Thomson's theory, knots such as the ones in figure 1a (the *unknot*), figure 1b (the *trefoil knot*) and figure 1c (the *figure eight knot*) could, in principle at least, model atoms of increasing complexity, such as the hydrogen, carbon, and oxygen atoms, respectively. For knots to be truly useful, however, mathematicians searched for some precise way of proving that what appeared to be different knots (such as the trefoil knot and the figure eight knot) were really different—they couldn't be transformed one into the other by some simple manipulation. Towards the end of the nineteenth century, the Scottish mathematician Peter Guthrie Tait and the University of Nebraska professor Charles Newton Little published complete tables of knots with up to ten crossings. Unfortunately, by the time that this heroic effort was completed, Kelvin's theory had already been totally discarded as a model for atomic structure. Still, even without any other application in sight, the mathematical interest in knot theory continued at that point for its own sake. The only difference was that, as the British mathematician Sir Michael Atiyah has put it, "the study of knots became an esoteric branch of pure mathematics."

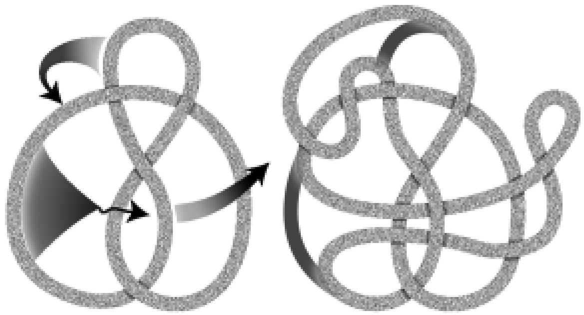

Figure 2: Deforming a knot. Image created by Ann Feild.

One of the main goals of knot theory has always been to identify properties that truly distinguish knots—to find what are known as *knot invariants*. A knot invariant acts very much like a "fingerprint" of the knot; it does not change by superficial deformations of the knot (for example, of the type demonstrated in figure 2). At first blush, you may think that the minimum number of crossings in a knot could serve as such an invariant. After all, no matter how hard you try, you will never be able to reduce the number of crossings of the trefoil knot (figure 1b) to fewer than three. However, the minimum number of crossings is actually not a very useful invariant. As figure 1 demonstrates, there are three different knots with six crossings and no fewer than seven different knots with seven crossings.

Two major breakthroughs in knot theory occurred in 1928 and in 1984. In 1928, the American mathematician James Waddell Alexander discovered an algebraic expression (known as the *Alexander polynomial*) that uses the arrangement of crossings to label the knot. For example, the Alexander polynomial for the trefoil knot is t^2-t+1. Two knots that have different Alexander polynomials are indeed different (e.g., the Alexander polynomial for the figure eight knot is t^2-3t+1). Unfortunately, two knots that have the same Alexander polynomial may still be different. Consequently, while it was certainly very useful, the Alexander polynomial was still not perfect for classifying knots. Decades of work in the theory of knots finally produced the second breakthrough in 1984. The New Zealander-American mathematician Vaughan Jones detected an unexpected relation between knots and another abstract branch of mathematics (known as *von Neumann algebras*). This led to the discovery of a more sensitive invariant than the Alexander polynomial, which became known as the *Jones polynomial*. The Jones polynomial distinguishes, for instance, even between knots and their mirror images (figure 3), for which the Alexander polynomials were identical.

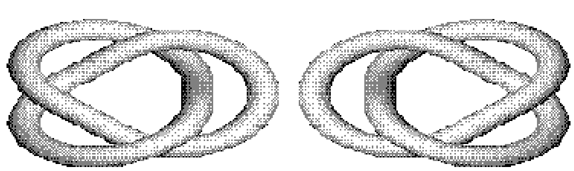

Figure 3: A knot and its mirror image. Image created by Ann Feild.

More importantly, the flurry of activity that ensued following Jones's discovery suddenly connected a bewildering variety of areas in mathematics and physics, and penetrated even into string theory—the current most promising attempt to reconcile general relativity with quantum mechanics.

In particular, string theorists Hirosi Ooguri and Cumrun Vafa discovered that the number of complex topological structures that are formed when many strings interact is related to the Jones polynomial. Furthermore, the leading string theorist Ed Witten demonstrated that the Jones polynomial affords new insights in one of the most fundamental areas of research in modern physics, known as *quantum field theory*.

Knots leading the way, from the atom to pure maths and back to physical matter.

The lesson from this very brief history of knot theory is remarkable. First, it was the *active* effectiveness of mathematics that came into play. Physicists needed a model for the atom, and when knots appeared to provide the appropriate tool, a mathematical theory of knots took off. When a better mathematical model (in the form of the Bohr atom) was discovered, mathematicians did not abandon knot theory. Driven only by their curiosity, they continued to explore the properties of knots for many decades. The mere possibility of understanding the properties of knots and the principles that govern their classification was seen by most mathematicians as exquisitely beautiful and essentially irresistible. However, then came the surprising *passive* effectiveness of mathematics. Unexpectedly, the Jones polynomial and knot theory in general turned out to have wide-ranging applications in string theory.

What makes this story even more striking is the following fact. Recall that Thomson started to study knots because he was searching for a theory of atoms, then considered to be the most basic constituents of matter. By a remarkably circular twist of history, knots are now found to provide answers in string theory, our present-day best effort to understand the constituents of matter! So knot theory emerged from an attempt to explain physical reality, then it wandered into the abstract realm of pure mathematics—only to eventually return to its ancestral origin. Isn't this absolutely amazing?

history of mathematics philosophy of mathematics knot knot theory logic axiom infinity public understanding of mathematics mathematics and art proof

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About this article

Mario Livio's book *Is God a Mathematician?* is scheduled to appear on January 6, 2009 and reviewed in this issue of *Plus*. It follows the lives and thoughts of some of the greatest mathematicians in history, and attempts to explain the "unreasonable effectiveness" of mathematics.

Dr Mario Livio is a senior astrophysicist at the Space Telescope Science Institute (STScI), the institute which conducts the scientific program of the Hubble Space Telescope. He received his PhD in theoretical astrophysics from Tel Aviv University in Israel, was a professor in the Physics Dept. of the Technion-Israel Institute of Technology from 1981 till 1991, and joined STScI in 1991. Dr Livio has published over 400 scientific papers and received numerous awards for research and for excellence in teaching. His book, *The Golden Ratio*, won him the *Peano Prize* and the *International Pythagoras Prize* in 2004.

His interests span a broad range of topics in astrophysics, from cosmology to the emergence of intelligent life. Dr Livio has done much fundamental work on the topic of accretion of mass onto black holes, neutron stars, and white dwarfs, as well as on the formation of black holes and the possibility to extract energy from them. During the past decade, Dr Livio's research focused on supernova explosions and their use in cosmology to determine the nature of the *dark energy* that pushes the universe to accelerate, and on extrasolar planets.