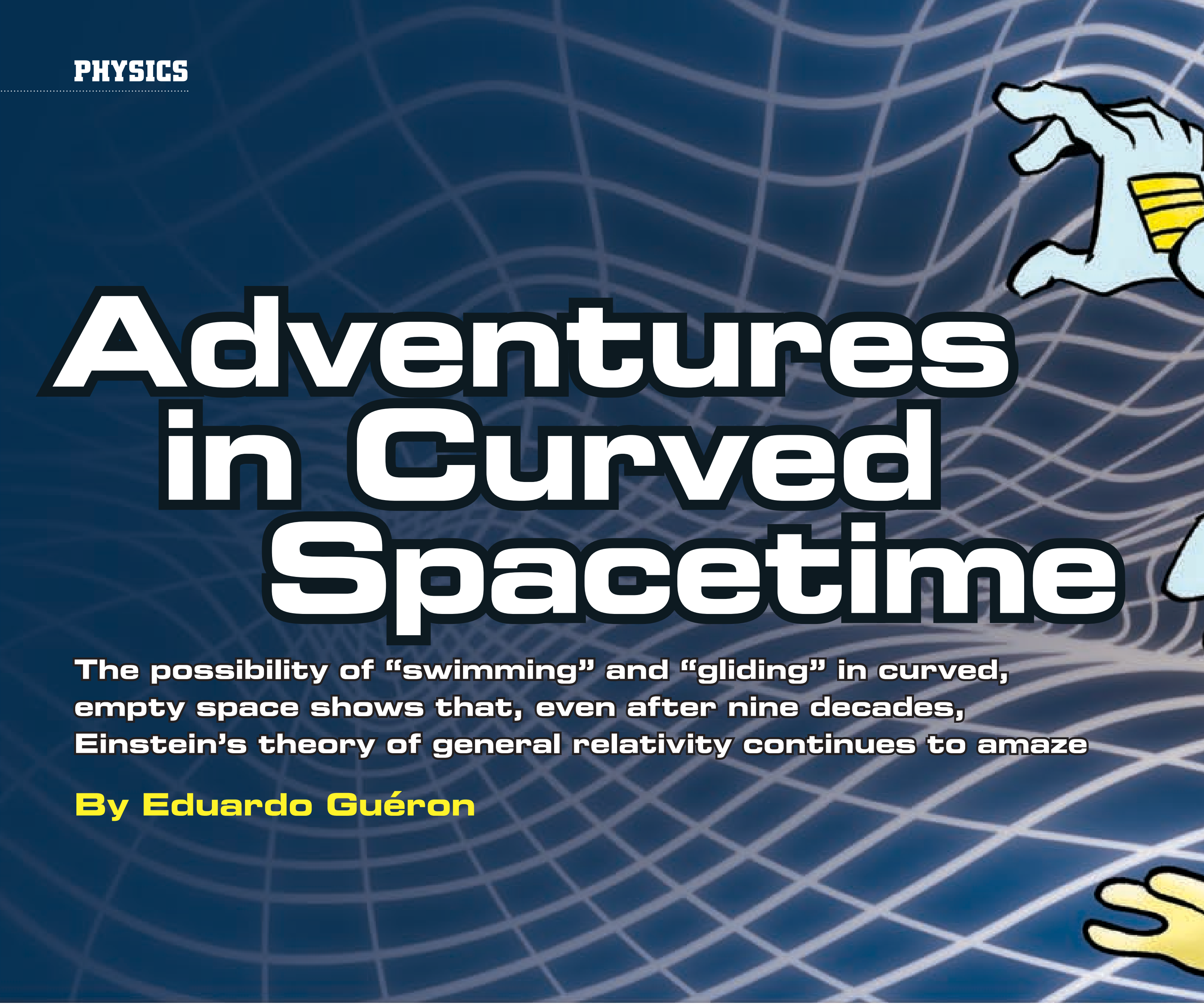

Adventures in Curved Spacetime

The possibility of "swimming" and "gliding" in curved, empty space shows that, even after nine decades, Einstein's theory of general relativity continues to amaze

By Eduardo Guéron

KEY CONCEPTS

- In Albert Einstein's theory of general relativity, gravity arises from spacetime being curved. Today, 90 years after Einstein developed the theory's equations, physicists are still uncovering new surprises in them.
- For example, in a curved space, a body can seemingly defy basic physics and "swim" through a vacuum without needing to push on anything or be pushed by anything.
- Curved spacetime also allows a kind of gliding, in which a body can slow its fall even in a vacuum. *—The Editors*

In a famous series of stories in the 1940s, physicist George Gamow related the adventures of one Mr. C.G.H. Tompkins, a humble bank clerk who had vivid dreams of worlds where strange physical phenomena intruded into everyday life. In one of these worlds, for instance, the speed of light was 15 kilometers per hour, putting the weird effects of Einstein's theory of special relativity on display if you so much as rode a bicycle.

Not long ago I figuratively encountered one of Mr. Tompkins's great grandsons, Mr. E. M. Everard, a philosopher and engineer who is carrying on his ancestor's tradition. He told me of an amazing experience he had involving some recently discovered aspects of Einstein's theory of general relativity, which I will share with you. His remarkable story is replete with curved spacetime, cats twisting in midair, an imperiled astronaut dog paddling through a vacuum to safety—and Isaac Newton perhaps spinning in his grave.

Dangerous Curves Ahead

In a far-off region of the cosmos, Mr. Everard had gone outside his spaceship to repair an errant antenna. He noticed that the beautiful lights of the distant stars looked distorted, as though he were viewing them through a thick lens. He felt, too, something gently stretching his body. Suspecting he knew what was afoot, he took a laser pointer and a can of shaving cream from his utility belt and turned on his jet pack to test his idea.

With the laser beam serving as a guide, he jetted straight out 100 meters, turned left to travel several dozen meters in that direction and finally returned to his starting point, drawing a

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triangle of foam like a cosmic skywriter. Then he measured his triangle's vertex angles with a protractor and added them up. The result was more than 180 degrees.

Far from being nonplussed by this apparent violation of the rules of geometry, Mr. Everard fondly remembered a mischievous non-Euclidean incident in his childhood, when he drew triangles on the globe in his parents' study. There, too, the angles added up to more than 180 degrees. He concluded that the space around him also must be curved much like the surface of that globe, so many years and light-years away. The curvature would account for the distorted starlight and the slightly unpleasant feeling of being stretched.

Thus, Mr. Everard understood he was experiencing textbook effects of general relativity. Experiments of a rather more refined nature than his jaunting about with shaving cream had confirmed these effects long ago: matter and energy cause space and time to curve, and the curvature of spacetime causes matter and energy (such as his laser beam and the light from the stars) to follow curved trajectories. His feet and his head "wanted" to follow slightly different curves, and the discrepancy produced the stretching sensation.

Musing on these facts, Mr. Everard pressed the button to engage his jet pack again to return to his spaceship—and nothing happened. Alarmed, he saw his fuel gauge was at zero, and he was a good (or rather, bad) 100 meters from the safety of his air lock. In fact, he and his triangle of foam were drifting away from his spacecraft at a constant velocity.

Acting quickly, he flung his protractor, laser,

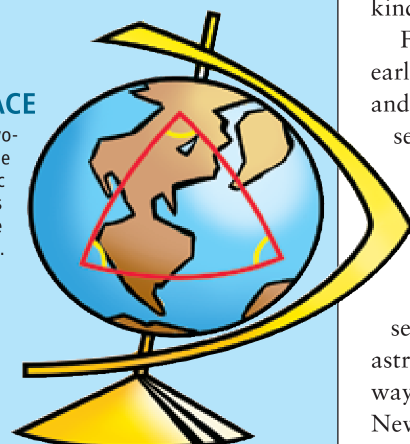
[BASICS]

CURVED SPACETIME

General relativity describes gravity as arising from the curvature of spacetime, but what does it mean for spacetime to be curved and what are some of the consequences?

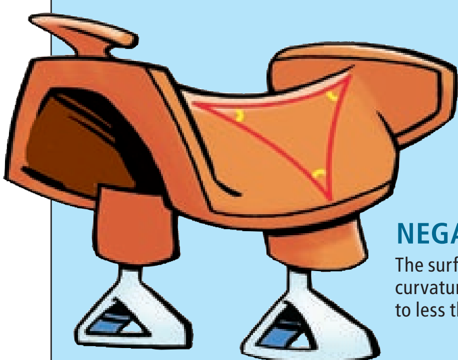
FLAT SPACE

The geometry taught in school is Euclidean, or the geometry of "flat" space. In such a space, the angles of a triangle add up to 180 degrees. A two-dimensional plane, such as the surface of a billiards table, is a flat space. To a very good approximation, so, too, is the three-dimensional world around us: if you "draw" a triangle in the air using three laser beams to mark the sides, the angles will add up to 180 degrees anywhere that you draw it.



CURVED SPACE

The surface of a sphere is an example of a curved two-dimensional surface. On a sphere, the angles of a triangle add up to more than 180 degrees, which is a characteristic of a region with "positive" curvature. The triangle's sides may look curved to us in three dimensions, but they are perfectly straight to an ant crawling across the sphere.

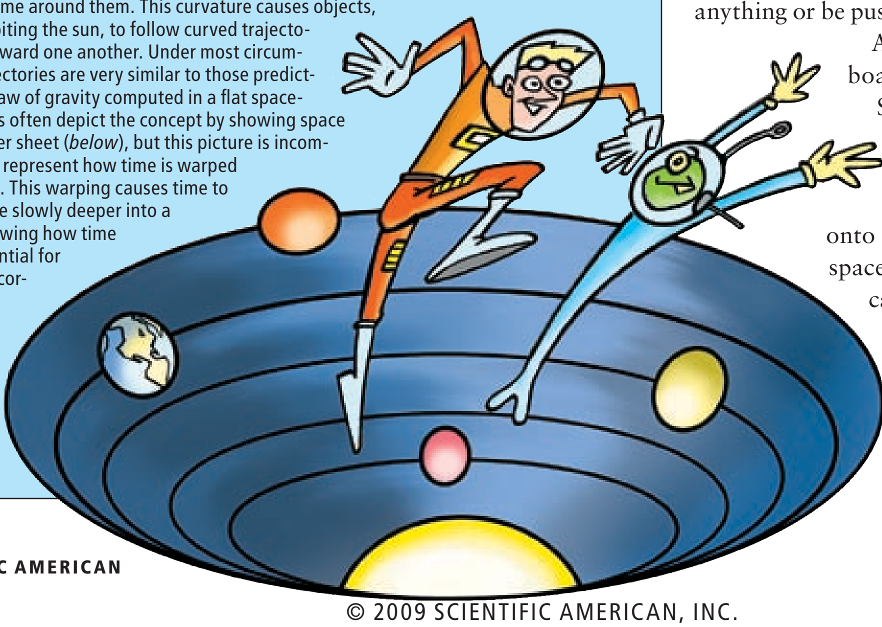


NEGATIVE CURVATURE

The surface of a saddle shape has negative curvature—the angles of a triangle add up to less than 180 degrees.

GRAVITY COMES FROM CURVATURE

According to general relativity, concentrations of mass and energy curve the spacetime around them. This curvature causes objects, such as Earth orbiting the sun, to follow curved trajectories and to fall toward one another. Under most circumstances, the trajectories are very similar to those predicted by Newton's law of gravity computed in a flat spacetime. Illustrations often depict the concept by showing space as a curved rubber sheet (*below*), but this picture is incomplete; it does not represent how time is warped along with space. This warping causes time to pass slightly more slowly deeper into a gravity well. Knowing how time is warped is essential for determining the correct trajectories.



can of foam and all the other items on his utility belt directly away from his spacecraft. In accord with the principle of momentum conservation, with each throw he recoiled a little in the opposite direction—toward his ship. He even unharnessed his jet pack and shoved that dead weight away as forcefully as he could. Alas, when he had nothing left to hurl, he found he had done only enough to counteract his initial motion away from the ship. He was now floating motionless with respect to his ship but still far away from it. His situation may have seemed hopeless: his high school physics teacher had impressed on him that it is not possible to accelerate a body without an external force or some kind of mass ejection.

Fortunately for our adrift friend, he had earlier established that he was in a curved space, and he was wise enough to know that some conservation laws in physics work differently in a curved space than in the flat (uncurved), Newtonian space of his school years. In particular, he remembered reading a 2003 physics paper in which planetary scientist Jack Wisdom of the Massachusetts Institute of Technology showed that an astronaut could move through curved space in ways that would be impossible according to Newton's laws of motion—simply by making the right movements with his arms and legs. In other words, he could swim. It did not require any fluid to push against; he could dog-paddle through the vacuum.

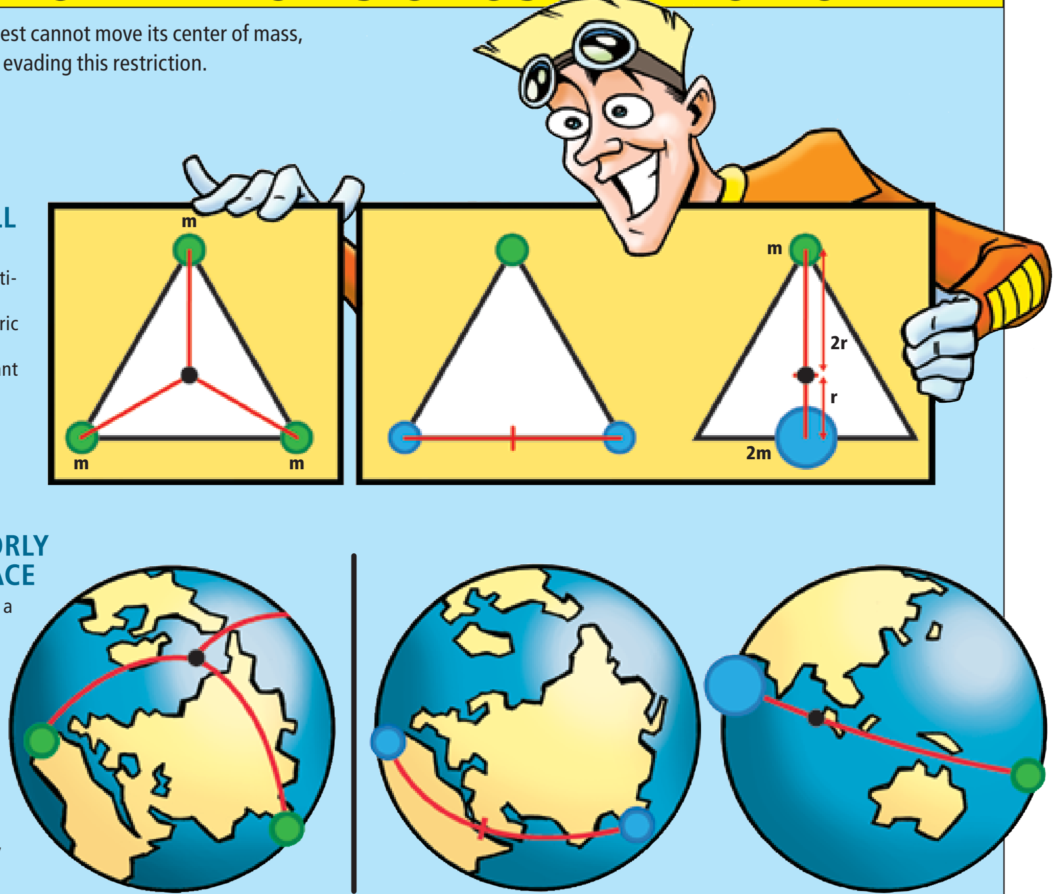
Wisdom's trick is rather like how a cat, dropped upside down, can twist its body and retract and extend its legs so that it flips over and lands on its feet. The laws of Newtonian mechanics permit the cat to change its orientation, but not its velocity, without needing to push on anything or be pushed by anything.

Astronauts such as those onboard the International Space Station use a version of the cat-twisting trick to turn around in weightlessness without needing to grab onto a handhold. In the curved spacetime of general relativity, a cat or an astronaut can pull off more impressive stunts. Our hero covered the distance back to his spacecraft in somewhat more than an hour—no Olympic record but certainly quick enough

[A LOOPHOLE]

WHY CURVATURE ALLOWS UNUSUAL MOTION

In flat space, an isolated system at rest cannot move its center of mass, but curved space has a loophole for evading this restriction.



CENTER OF MASS IS WELL DEFINED IN FLAT SPACE

Three balls of equal mass, *m*, at the vertices of an equilateral triangle have their center of mass at the triangle's geometric center (*black dot*). This position can be calculated as the point that is equidistant from all three corners (*left*), but it may also be calculated in two steps (*right*).

CENTER OF MASS IS POORLY DEFINED IN CURVED SPACE

Now imagine that the three balls are in a space curved like a surface of a sphere and are at locations corresponding to Dakar, Singapore and Tahiti on Earth. Computing the balls' center of mass by finding an equidistant point produces a location near the North Pole (*left*). Computing the center of mass in two steps, however, yields a spot near the equator (*right*). This ambiguity about the center of mass makes it possible to "swim" through a curved space.

to ensure that he would live to undertake more adventures.

Swimming Lessons

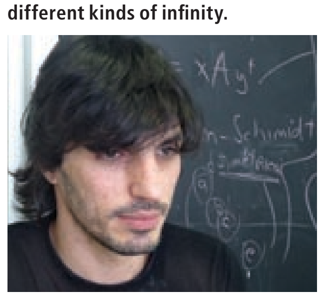
How exactly does Wisdom's phenomenon work? How is it an adventurer such as Mr. Everard can swim in space? In a flat space—the kind assumed by Newtonian mechanics and also special relativity—the center of mass of an isolated system (for example, astronaut plus dead jet pack) never accelerates. Suppose Mr. Everard had tied a long cord to his jet pack before he shoved it away and then reeled it back in. Throughout the entire exercise, as the jet pack and astronaut first moved farther apart and later came together again, the center of mass of the two would be unchanged. At the end, he and his jet pack would be back at their initial position. More generally, Mr. Everard cannot move merely by cyclically changing his shape or structure and then restoring it again.

In curved space, the situation is different. To understand why, imagine an alien creature with two arms and a tail, all of which it can extend and retract [*see box on page 43*]. To simplify the discussion, imagine that virtually all of the alien's mass is concentrated at the ends of its three limbs, a quarter of it in each hand and the other half at the tip of the tail. Floating in *flat* space, this alien is helpless. If it extends its tail by, say, two meters, the hands move forward one meter and the tail tip moves back one meter, maintaining the center of mass. Retracting the tail again brings the whole alien back to its starting position, just as with Mr. Everard and his inert jet pack. Similar things happen if the alien tries extending its arms. Whatever combination or sequence of limb extensions and retractions the alien carries out, its center of mass stays the same. The best it can do is use the cat trick (extend limbs, swing them around, retract them, swing them back again) to change the direction it is pointing.

But now imagine that this alien lives in a curved space, one shaped like the surface of a sphere. To help you picture it, I will use geo-

[THE AUTHOR]

Eduardo Guéron is associate professor of applied mathematics at the Federal University of ABC in Brazil (the "ABC Region" borders on the city of São Paulo). Guéron earned his Ph.D. at the State University of Campinas in Brazil in 2001, and he was a visiting scientist at the Massachusetts Institute of Technology from 2003 to 2004. He studies gravitation, dynamical systems and fundamental problems in general physics. Guéron also enjoys playing with his four-year-old daughter and trying to teach her that there are different kinds of infinity.

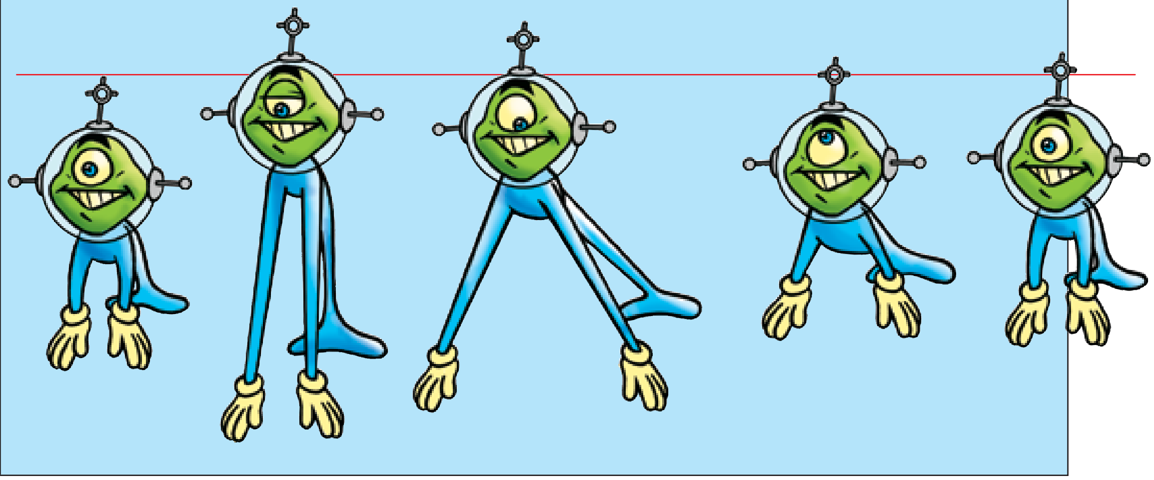


COURTESY OF EDUARDO GUÉRON (*Guéron*)

[HOW TO SWIM]

A SWIMMER IN CURVED SPACETIME

A tripodlike machine or alien creature could swim through empty but curved spacetime by cyclically extending, opening, retracting and closing its legs. Each cycle of four actions moves the tripod through space—here a small distance up the page—even though it ejects no propellant and no external force acts on it.



graphical terms to describe positions and directions on the sphere. The alien starts on the sphere's equator, its head pointing west and its arms and tail all retracted. It extends both arms, one to the north and one to the south. It then lengthens its tail while keeping its arms extended at right angles to its body. As in flat space, if the mass-laden tail tip moves one meter to the east, the hands move one meter west. Here is the crucial difference on the sphere: the alien keeps its arms aligned with the sphere's lines of longitude and the distance between those lines is greatest at the equator. Thus, when the alien's hands (nearer the north and south poles of the sphere) move one meter westward, its shoulders (on the equator) move more than one meter. Now when the alien retracts its arms, along the lines of longitude, it ends up with its hands more than one meter west. When it retracts its tail, restoring its original body configuration, it finds itself a short distance westward along the equator from its original position!

Spacetime is only very slightly curved, except near a black hole. So in practice you would be swimming for billions of years before you moved a millimeter.

By cyclically repeating these movements, the alien crawls along the equator. The unusually heavy tail tip and hands are not essential to the swimming; it is just easier to see how far the arms move in response to the tail stretching if all the mass is concentrated at those three points. And, as it happens, if the alien species depended on the curved-space swimming for its survival, it might evolve heavy knobs at its extremities to improve the efficiency of its swimming. After all, mass located at its elbows will not reach so far around the curvature of the sphere as its hands do and therefore will not produce as much extra movement of the body.

A sphere is a two-dimensional surface, but the same principle works in curved four-dimensional spacetime. Cyclic changes in the configuration of a system can lead to a net displacement. Wisdom's proposed swimmer was a tripod with telescoping legs. The legs can be retracted or extended in length, and the angle between them can be widened or narrowed. The tripod swims by extending its legs, spreading them, retracting them and closing them. The greater the curvature of spacetime where the tripod is, the farther it gets displaced by this sequence of moves.

Moving Violations?

Though surprising at first, swimming is a direct consequence of basic conservation laws, not a violation of them. Swimming works because the very concept of a center of mass is not well defined in a curved space. Suppose we have three one-kilogram balls located at the vertices of an equilateral triangle. On a flat surface, their center of mass is the geometric center of the triangle. You can calculate where the center of mass is located in a number of different ways, and each method gives the same result. You can find the point that is an equal distance from all three balls. Or you can replace two of the balls with a single two-kilogram ball located halfway between them and then calculate the center of mass of that ball and the third ball (the point one third of the way along the line to the third ball). The result will be the same. This geometric fact carries over into the dynamics of the system: the center of mass of an isolated system never accelerates.

On a curved surface, however, different computations may not give the same result. Consider a triangle formed by three equal-mass balls in Singapore, Dakar and Tahiti—all near the equator. A point equidistant to the three balls is near the North Pole. But if you replace the balls in Singapore and Dakar with a heavier one in between them and then calculate the position that is one third of the way along the great circle from that ball to the one in Tahiti, your answer will lie close to the equator. Thus, the “center of mass” on a curved surface is ambiguous. This geometric fact ensures that a system in a curved space can move even when it is isolated from any outside influences.

Other subtleties also arise. A standard physics homework assignment involves adding up the forces on a body to determine the net force. Physics students express forces as vectors, which are drawn as arrows. To add two vectors, they slide the arrows around so that the base of one arrow meets the tip of the other. In a curved space, this procedure has pitfalls: the direction of a vector can change when you slide it around a closed path. The procedure for calculating the total force on a body in curved space is therefore considerably more complicated and can result in oddities such as swimming.

Some effects in Newtonian gravitation may seem similar to spacetime swimming at first glance. For instance, an astronaut orbiting Earth could alter his orbit by stretching tall and curling into a ball at different stages. But these Newtonian effects are distinct from spacetime swimming—they occur because the gravitational field varies from place to place. The astronaut must time his actions, like a person on a swing does to swing faster. He cannot change his Newtonian orbit by rapidly repeating very small motions, but he *can* swim through curved spacetime that way.

That the possibility of spacetime swimming went unnoticed for nearly 90 years reminds us that Einstein’s theories are still incompletely understood. Although we are unlikely to construct a swimming rocket anytime soon, Nobel laureate physicist Frank Wilczek, also at M.I.T., has argued that Wisdom’s work raises profound questions about the nature of space and time.

In particular, Wisdom’s findings bear on the age-old question of whether space is a material

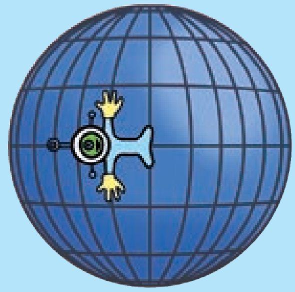
[WHY IT WORKS]

PADDLING ACROSS A SPHERE

Swimming through curved spacetime can be understood by considering a simpler two-dimensional alien swimmer that lives on the surface of a sphere.

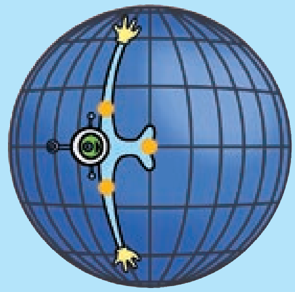
ON YOUR MARKS

The swimmer is facing west, with its arms pointing north and south and its tail pointing east. To simplify the discussion, imagine all of the swimmer’s mass is concentrated at the end of its limbs—a quarter in each hand and a half in the tip of its tail.



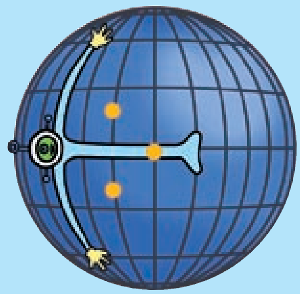
ARMS OUT

The arms stretch north and south (orange balls mark where the hands started). The equal and opposite movement keeps momentum balanced.



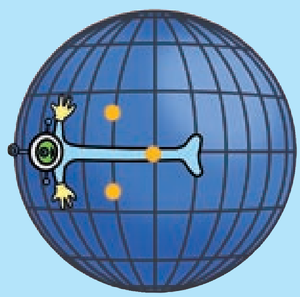
EXTEND TAIL

Now the tail stretches eastward. To balance momentum, the hands move west. Being near the poles of the sphere, the hands cross several lines of longitude to travel the same distance as the heavy tail tip, and the “shoulders” move a long way west along the equator.



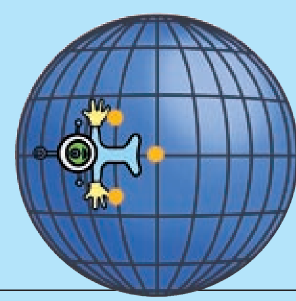
ARMS IN

The arms retract along the lines of longitude (which are the equivalent of straight lines on the sphere). The hands are now much farther west of their starting points than the tail tip is east of its starting point.



RETRACT TAIL

When the tail retracts again, the hands move back toward the east to balance momentum. The cycle of actions has moved the swimmer as a whole a short distance to the west because of the “extra” distance that the hands have traveled.



On a saddle shape, which has negative curvature, the same actions would move a swimmer eastward. See http://physics.technion.ac.il/~avron for animations of both examples.

"You cannot lift yourself by pulling on your bootstraps, but you can lift yourself by kicking your heels."

—Jack Wisdom, M.I.T.

object in its own right (a position known as substantivalism) or merely a convenient conceptual device to express the relations among bodies (a position known as relationalism) [see "A Hole at the Heart of Physics," by George Musser; SCIENTIFIC AMERICAN, September 2002].

To illustrate these viewpoints, imagine that Mr. Everard is floating in an otherwise empty universe. He would have no stars or galaxies to serve as reference points to judge his motion. Physicist and philosopher Ernst Mach, a relationalist, argued in 1893 that motion would be meaningless in this situation. Yet even a completely empty space can be curved, in which case Mr. Everard could swim through it. It therefore seems that spacetime acts as a virtual fluid against which the motion of an isolated body can be defined. Even completely empty space has a specific geometric structure—another point in favor of substantivalism. At the same time, though, matter (or any other form of energy) is what gives spacetime its geometric structure, so spacetime is not independent of its contents—a point in favor of relationalism. This debate, which crops up in the attempts to develop a unified theory of physics, remains unresolved.

On the Wings of Time

Worn out by the effort to swim back to his spaceship, Mr. Everard was resting inside the cabin and letting the autopilot plot a course back home. Suddenly, the alarm went off and the red lights started flashing, indicating that the spaceship was falling onto a massive planet. Mr. Everard was delighted by this opportunity for new and interesting discoveries, but landing on this planet would be a challenge. The ship had too little fuel for a powered descent, and the planet lacked an atmosphere, making a parachute useless.

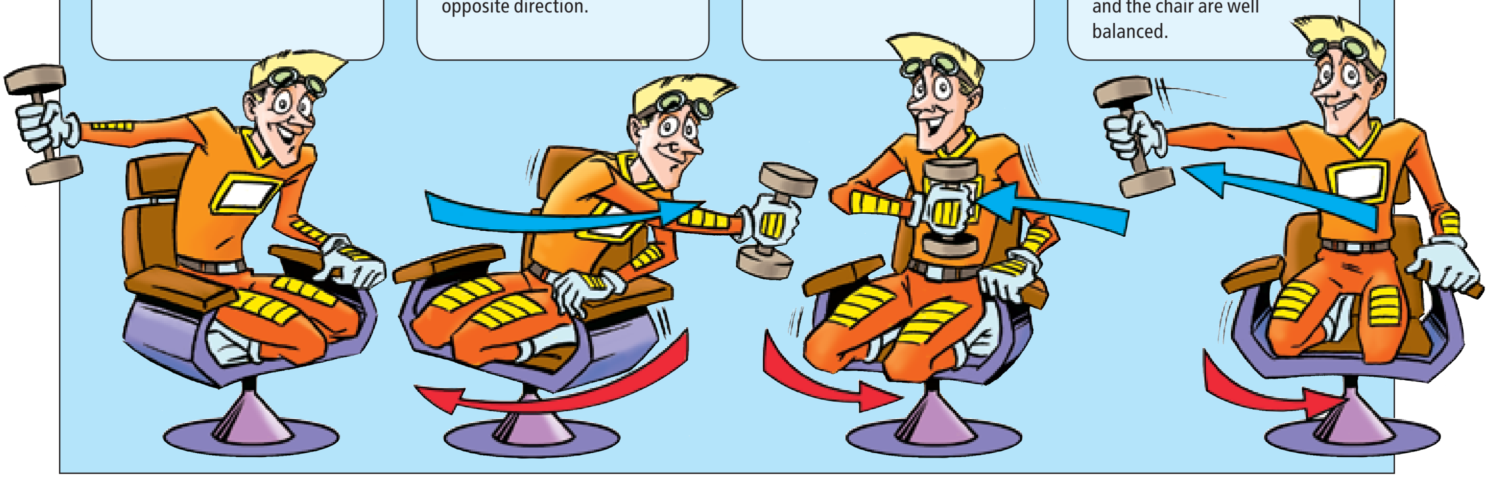
Fortunately, he remembered the 2007 paper that my colleague, mathematical physicist Ricardo A. Mosna of the State University of Campinas in Brazil and I wrote. Inspired by Wisdom's example, we came up with another way to exploit general relativity to control motion. Our analysis indicates that an object can slow its descent toward, say, a planet by repeat-

[DO-IT-YOURSELF]

TAKING IT FOR A SPIN

Near Earth, spacetime is so very close to being flat that you cannot "swim" through it to change your location. Yet you can change your orientation without needing an outside force (much like a falling cat twists to land on its feet). Here is one way to try it out. See www.ScientificAmerican.com for a video.

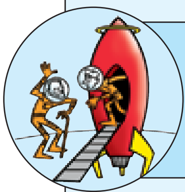
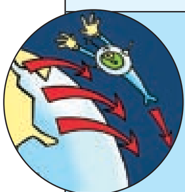
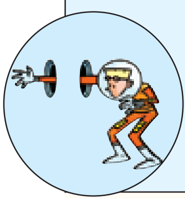
1. Kneel or sit on a swivel chair, preferably one that does not also rock. Hold a weight out to your side (the weight will increase the effect).
2. Swing the weight around to your other side, keeping your arm outstretched all the way. To conserve angular momentum, the chair (and you on it) should swivel in the opposite direction.
3. Now bring the weight back across you, keeping it as much as possible on a path going through the axis of the chair.
4. The chair will turn back part of the way to where it began, but you should end up rotated from your original position. Repeating the move can turn you full circle if you and the chair are well balanced.



[OTHER ODDITIES]

BEYOND NEWTON

General relativity has long predicted several effects that have no analogue in Newtonian gravitation, in addition to the recently uncovered phenomena of spacetime swimming and gliding.

EFFECT	EXAMPLE	EXPLANATION	THEORY	STATUS
 Gravitational time dilation	A person travels near a black hole; she returns younger than her stay-at-home twin	Time passes more slowly in a strong gravitational field	Inferred by Albert Einstein while developing general relativity	Used in technology: the Global Positioning System (GPS) has to allow for gravitational time dilation in the timing of its signals to compute accurate positions
 Gravitational waves	Waves of gravity propagating out from a binary star system at the speed of light	Gravitational waves are traveling oscillations of spacetime geometry, as if spacetime itself were undergoing vibrations of compression and expansion	General relativity's equations clearly permit waves, but the waves are hard to analyze exactly	Observed indirectly in the late 1970s: the orbital period of a pulsar and a neutron star forming a binary system became shorter over time as predicted to occur because of gravitational wave emission. LIGO and other experiments are seeking direct observations of gravitational waves
 Lense-Thirring effect	A satellite near Earth feels a force pulling it in the direction of Earth's rotation	Like a ball spinning in molasses, a rotating mass drags spacetime itself around a small amount	Predicted by Joseph Lense and Hans Thirring in 1918	In February 2009 researchers announced that results from the Gravity Probe B satellite matched the prediction within an experimental uncertainty of 15 percent
 Wormholes	A hypothetical shortcut connecting two different regions of the universe	Extraordinary hypothetical kinds of energy would provide negatively curved spacetime, which is required to form a wormhole structure	Discussed as early as 1916; researchers showed that general relativity's equations permit traversable wormholes in 1988	Still very speculative; most physicists believe that they will never be found

edly stretching and contracting in an asymmetric fashion—meaning the extending motion is faster than the retracting. A ship equipped with a device moving in that fashion could act as a glider even in the absence of air.

In this case, the effect has to do with the temporal rather than spatial qualities of the motion, which brings to light one of the deepest aspects of Einstein's theories: the connection between space and time. In Newtonian mechanics, physicists can specify the location of events using three coordinates for spatial position and one for the time, but the concepts of space and time are still distinct. In special relativity, they are inextricably intertwined. Two observers with different velocities may not agree on their measurements of the distance or time interval between two events, but they do agree on a certain amalgam of space and time. Thus, the observers see time and space, considered separately, differently—yet see the same spacetime.

In general relativity, the structure of spacetime becomes distorted (that is, curved), producing what we perceive as the force of gravity. Whereas Newtonian gravity involves only space, relativistic gravity also involves time. This distortion of both space and time leads to effects such as one known as frame dragging: a rotating body (such as Earth) exerts a slight force in the direction of its rotation on other nearby objects (such as orbiting satellites). Loosely speaking, the spinning Earth drags spacetime itself around slightly. More generally, the velocity of motion of a mass influences the gravitational field it produces. Frame dragging and the glider are both examples of this phenomenon.

The swimming effect arises from non-Euclidean geometry, and the relativistic glider is a consequence of indissolubility of space and time. Other such phenomena may remain to be recognized and understood within the inscrutable equations of general relativity. Mr. Everard and other disciples surely have more adventures in store. ■

MORE TO EXPLORE

Space, Time, and Gravity: The Theory of the Big Bang and Black Holes. Robert M. Wald. University of Chicago Press, 1992.

Swimming in Spacetime: Motion in Space by Cyclic Changes in Body Shape. Jack Wisdom in *Science*, Vol. 299, pages 1865–1869; March 21, 2003.

Swimming versus Swinging Effects in Spacetime. Eduardo Guéron, Clóvis A. S. Maia and George E. A. Matsas in *Physical Review D*, Vol. 73, No. 2; January 25, 2006.

Relativistic Glider. Eduardo Guéron and Ricardo A. Mosna in *Physical Review D*, Vol. 75, No. 8; April 16, 2007.