# Notice

**The attached document is undergoing radical revision**

**Computer analysis by Joe Brophy on about 70,000 puzzles revealed that most puzzles can be solved by a few simple rules.**

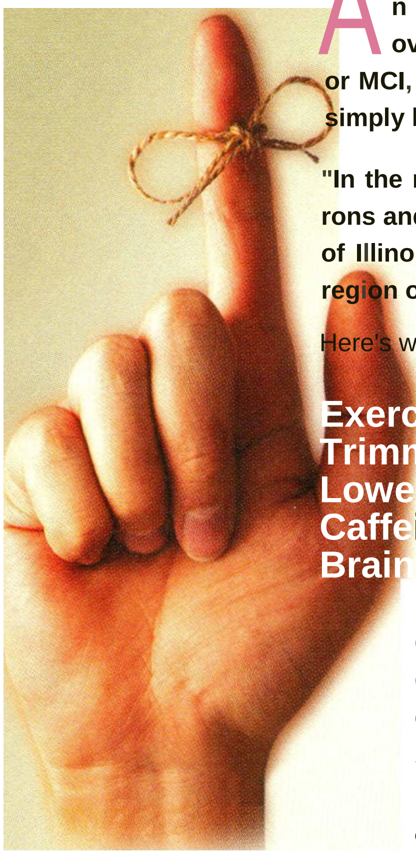
**The new document to be released shortly will contain**

1. **About 100,000 puzzles and solutions**
2. **Many of the puzzles will contain step by step solutions to demonstrate how difficult puzzles can be solved simply by using the "only choice" rule and true/false chains**
3. **A comprehensive list of techniques used by skilled players, most of can be replaced by simple "only choice" and true/false chains.**

# You Must Remember

## How to keep your brain young

### Nutrition Action: April 2009, www.espinet.org



**An estimated one out of four people in their 80s-and one out of three over 80 has dementia! Many more have "mild cognitive impairment," or MCI, which often means that dementia is on its way. But most of us are simply losing our edge.**

**"In the normal brain, there's shrinkage of the dendrites that connect neurons and there's less production of neurotransmitters," says the University of Illinois's Arthur Kramer. "And the hippocampus, which is the memory region of the brain, gets smaller."**

Here's what might-and might not-maintain your brain wiring as you get older.

**Exercise**
**Trimmer Waist**
**Lower Blood Pressure**
**Caffeine curbs Alzheimer's Plaque**
**Brain Training**

- **maintaining friendships**
- **daily conversation**
- **getting outside your home**
- **crossword puzzles**
- **sudoku**
- **playing a musical instrument**
- **artistic hobbies**

# Senior Sudoku


**Adventures in Learning**

**Colby Sawyer College, Fall 2009**

**ILEAD, Fall 2010**

**Joseph T. & Carole A. Brophy**




**download**

**http://www.brophy.net/sudoku/ParsimoniousForce.pdf**

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## Why Wherefore this Compendium?

The author, Joseph T. Brophy is preparing to teach classes on Sudoku for Senior Citizens at various Adventures in Learning programs sponsored by local colleges and universities in New Hampshire. **This compendium is placed in the public domain,** with the understanding that the contents are not used for commercial purposes. Every effort has been made to give credit where credit is due; and to honor copyright restrictions. If a copyright has been violated, please contract josephbrophy@brophy.net and the violation will be immediately corrected.

This document is not for sale. Any revenues otherwise obtained from the sale of this compendium should be sent as a donation to **Adventures in Learning, Colby-Sawyer College, and New London, NH 03257**.

## Quick Launch!

**net, net, net for geniuses; this is repeated on page 50 in more readable form for the rest of us.**

1. **There are two types of players: beginners and advanced players**
2. **Beginners should use 2 or 3 simple techniques (i.e. forces) that will enable them to solve simple and moderate puzzles. Examples: Cross Hatching; Group Cross Hatching, Claims, Visualization, Muddling, Naked Singles, The Force, The Complete, Only Choice, Last Remaining Cell, Hidden Singles, Slicing and Dicing, etc. The principles underlying all of these techniques are very similar; hopefully, once understood, we can forget all the buzzwords.**
3. **Advanced players should use Chains of all sorts to solve the most difficult problems. Example: Swordfish, Squirmbag, X-Wing, Y-Wing, XY-Wing, Uniqueness, Gordonians, Eppstein Bi-Value Cycle, True False Chain, Naked Quads. The principles underlying these chains are all similar. Learn the principles and forgot all the buzzwords.**
4. **The remainder of Sudoku literature is essentially gobblygook: a collection of hard to remember patterns, terms and suggested rules that are probably best forgotten. Unfortunately, it is necessary to learn the tools and patterns used by experienced players. This will lead to a more thorough understanding of the simplicity of chains. We will cover all the patterns mentioned in the following sections. If you are patient, jump to this section, repeated on page 50.**
5. **What are the biggest pitfalls in Sudoku? Clerical and logical errors using the various methods of identifying potential values within each cell. Clerical and logical errors happen all the time when one is momentarily distracted for just a split second; that's all it takes; let's call it a "senior moment." Nothing is more disheartening than discovering an inconsistency in the End Game caused by a clerical error or a logical error.**
6. **To solve the more difficult puzzles, one has to increasingly rely on one's short term memory, and remember lots of data points and their relationships to each other within Sudoku Units. Each cell is logically connected to 20 other cells forming a Sudoku Unit. The value in a given cell cannot be repeated in any of the other 20 cells in the Sudoku Unit.**
7. **Although there are only 81 cells in a regular Sudoku, there could be as many as 200 to 400 potential values in play. This is lot of values and relationships to remember!**
8. **This leads to the question of how can one proceed systematically and efficiently to keep track of all the data points and their relationships? This requires a methodology or Markup System. Most good players develop their own system. We will discuss various systems in the following pages.**
9. **One of the major decisions when solving Sudoku is: when should one begin to markup the detail potential candidates for each empty cell? Identifying all the potential values in all empty cells is hard, time consuming work, and error prone. If one is 99% accurate in a markup, then they will surely make 3 mistakes (on average), either omissions or commissions. It only takes one mistake to riddle the puzzle with inconsistencies in the End Game.**
10. **Delaying the detail markup as long as possible is a winning strategy. Alternatively, one might consider using a software program like Angus Johnston's "Simple Sudoku" to fill in all potential values and/or provide hints with explanations. For the venturesome, the program runs only on PCs and can be found at www.angusj.com/sudoku.**

## What is Sudoku?

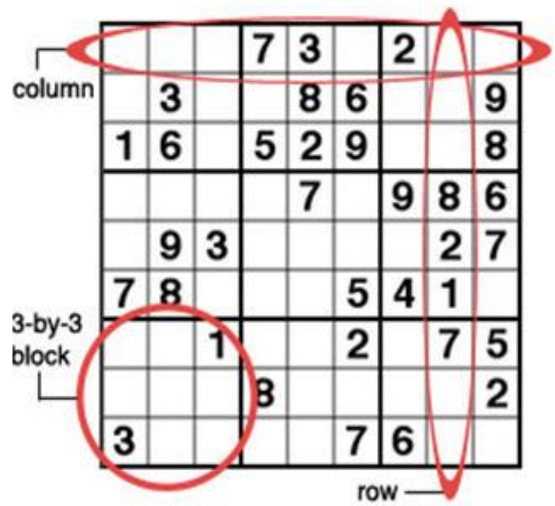
**Sudoku** is a **symbol pattern puzzle**; it is not really a numerical puzzle, as you might suppose. Sudoku consists of a cell grid of 9 cells x 9 cells to be filled in with **numbers** **1** to **9**. Actually though, the cell entries need not be numbers. You could just as well fill the cells with **letters** (e.g. a,b,c,d,e,f,g,h,i), or **colors** (of the rainbow: red, orange, yellow, green, blue, indigo, violet, and black and white to make up nine unique entries), or **shapes** (cell, triangle, circle, ellipse, rectangle, pentagon, hexagon, octagon and parallelogram) - or anything as long as they form a distinct and meaningful pattern for the solver. The goal of the puzzle is to place these symbols in the 81 cells in the grid while obeying **one simple rule as follows:**

There is but just **one simple rule** controlling where you can place numbers or symbols in the grid. If you stare at a 9x9 Sudoku Grid, you will observe that there are **9 Columns, 9 Rows and 9 Boxes** (also called Regions or Blocks).

Each Row, Column, Box is called a **Group** (or an Area) for a total of 27 Groups (or Areas). We will use the definitions Group and Box hereinafter. **The simple rule is that a symbol must occur once and only once in each of the 27 Groups of nine cells**. Cells are also called Squares.

The illustration on the right along with three other illustrations below of the same ilk (in the Middle English sense of the word rather than the pejorative sense), were found in the **Christian Science Monitor** Website/HomeForum/Sudoku.

**JUST TO REPEAT: The simple rule is that a symbol must occur once and only once in each of the 27 Groups of nine cells**.




Such a simple rule leads to all the amazing variability of the more than 5 billion (**5,472,730,538)** unique Sudoku puzzles according to Professor Frazer Jarvis, University of Sheffield, see Wikipedia/Sudoku. But it doesn't stop there; it even gets better.

Let us show you **WHY one Sudoku puzzle goes a long way**!

**Sudoku versus Sodoku?** Caution; as an aside before proceeding -

Make sure you get the spelling correct.

According to Wikipedia, **Sodoku** is a bacterial zoonotic disease. It is caused by outnumbered Spirochete Spirillum minus. It is a form of rat-bite fever caused by rat bites or scratches. Mostly seen in Asia.

**One Sudoku Puzzle goes a Long Way**: Did you ever complete a Sudoku puzzle that you really enjoyed; and wished you could find more puzzles like it. Here is a trick that will also provide you with a deeper understanding of the nature of a Sudoku puzzle.

The numbers or symbols in a particular game can be interchanged with each other. For example, you can make all the 9's = 5, and all the 5's = 3, and all the 3's = 9, and you have a new puzzle, that is really the same as the original puzzle. In addition, the columns and rows also can be interchanged as long as they stay within a Box alignment. Remember there are 3 Rows of Boxes and 3 Columns Boxes. A Column of 3 Boxes can be interchanged with another Column of Boxes. Similarly a Row of Boxes can be interchanged with another Row of Boxes. Believe it or Not! A single Sudoku puzzle can be changed into **more than 1.2 trillion different puzzles**. All 1.2 trillion puzzles may look different but are numerically or symbolically identical and will yield the same solution. **Wow!**

**Skip the following unless you are a math enthusiast and want more detail**.

Skip and Go to the paragraph "Origins of Sudoku."

**For the Math Enthusiast; *look at the following four Puzzles***: The following four puzzles may look different but they are really identical and only disguised by transformations of rows or columns or boxes. The optics have changed, they have been disguised; but not the algorithm underlying their solution. The puzzle in the upper left corner is one of about 1.2 trillion Sudoku puzzles that are absolutely identical to each other.

1. If you rotate the red puzzle 90° CCW, you get the blue puzzle.
2. If you swap columns 1 and 2 in the blue puzzle, along with a few other column and Box swaps, you get the green puzzle.
3. If you rotate the green puzzle 90° CW, and **add 4 modulo 9** to each of its numbers, you get the pink puzzle. **4 modulo 9 is short for add 4 to the given number to obtain a new number, and if the new number is greater than 9, then subtract 9 to get an another new number that must either 1,2,3, or 4.**
4. CHECK: The answers are on page 48. The solution involves all major techniques.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | 8 | 2 | | 1 |
| 2 | 1 | | | | 7 | | | 5 |
| 5 | 6 | | | 1 | | | | |
| | 8 | 9 | | | | 5 | | |
| | | 6 | | | | 4 | | |
| | | 5 | | | | 9 | 3 | |
| | | | | 9 | | | 7 | 4 |
| 8 | | | 7 | | | | 9 | 3 |
| 9 | | 1 | 3 | | | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 5 | | | | | 4 | 3 | |
| | | | | | 3 | 7 | 9 | |
| 2 | | | 5 | 4 | 9 | | | |
| 8 | 7 | | | | | | | |
| | | 1 | | | | 9 | | |
| | | | | | | | 7 | 3 |
| | | | 9 | 6 | 5 | | | 1 |
| | 1 | 6 | 8 | | | | | |
| | 2 | 5 | | | | | 8 | 9 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5 | 1 | | 4 | | 3 | | | |
| | | | 7 | | 9 | | 3 | |
| | 2 | | | | | 4 | 9 | 5 |
| 7 | 8 | | | | | | | |
| | | 1 | 9 | | | | | |
| | | | | 3 | 7 | | | |
| | | | | 1 | | 6 | 5 | 9 |
| 1 | | 6 | | | | | | 8 |
| 2 | | 5 | | 9 | 8 | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 6 | 5 | | | | 2 | | | 9 |
| | | | | | 3 | 6 | | 5 |
| 9 | 1 | | | 5 | | | | |
| | | | | 4 | | | 2 | 8 |
| 4 | | 5 | 7 | | | | | |
| 3 | | | 2 | | | | 4 | 7 |
| | | 1 | | | | 8 | | |
| | | 9 | | | | 4 | 7 | |
| | 3 | 4 | | | | 9 | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| I | E | | | | | F | G | |
| | | | | | G | C | A | |
| H | | | E | F | A | | | |
| B | C | | | | | | | |
| | | I | | | | A | | |
| | | | | | | | C | G |
| | | | A | D | E | | | I |
| | I | D | B | | | | | |
| | H | E | | | | | B | A |

This is a legitimate puzzle using the letters ABCDEFGHI. It is patterned after a diabolical puzzle found in Michael Mepham's ***"The Book of Sudoku."*** Mepham's puzzles are of the highest quality and follow another of the Golden Rules: **The numbers must be placed in a Symmetrical pattern**. By the way, in Mepham's puzzle, A=1, B=2, C=3, D=4, E=5, F=6, G=7, H=8 and I=9.

The puzzle and solution are provided elsewhere in this document.

**HERE IS THE LESSON**: This pattern of letters can yield millions of puzzles. The letter A can be assigned any number from 1 to 9, Similarly for B, C, thru I. As long as the assignments are unique, we are OK.

This yields 9x8x7x6x5x4x3x2x1 unique permutations which is = 9! or 9 Factorial which = 362,880 different puzzles.

Each puzzle will look different, but their solutions will be very similar, and algorithmically identical.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | B | H | | I |
| H | I | | | | C | | | E |
| E | D | | | I | | | | |
| | B | A | | | | E | | |
| | | D | | | | F | | |
| | | E | | | | A | G | |
| | | | | A | | | C | F |
| B | | | C | | | | A | G |
| A | | I | G | | | | | |

This puzzle is identical to the above but has been rotated 90° clockwise. Puzzles can be rotated, flipped on their horizontal, vertical or diagonal axis to look different even though they are identical.

Furthermore, columns 1, 2 and 3 can be interchanged without changing the identity of the puzzle. The same for columns 4, 5 and 6, and the same for columns 7, 8 and 9.

This hold true for rows 1,2,3 and rows 4,5,6 and rows 7,8,9

Furthermore the columnar Boxes 1,4,7 can be swapped with Boxes 2,5,8 or Boxes 3,6,9 respectively.

The same rationale for the row Boxes 1,2,3 and 4,5,6 and 7,8,9.

The important requirement is that you **cannot break up a Group and interchange** its cells with another Group.

## Math Review:

The simple puzzle pattern in the first illustration above is solvable. Any of its letters ABCDEFGHI can be assigned to any of the numerals 123456789, so long as each letter-number assignment is unique, producing 9! = 362,880 different puzzles with different number combinations. Each puzzle, however, is identical in structure. The solution algorithm may differ because of the patterns of numerals may hide or provide better optics for clues, but all will lead to the same solution.

Each of the 3 columns within a columnar Box group can be interchanged in 6 ways, for a total of 6x6x6 combinations for the 3 sets of 3 columns.

Each of the 3 rows with a row Box group can be interchanged in 6 ways, for a total of 6x6x6 combinations for the 3 sets of 3 rows.

Each of the 3 columnar Box groups can be interchanged with the two other columnar Box groups in 6 ways.

Each of the 3 row Box groups can be interchanged with the two other row Box groups in 6 ways.

This leads to 6x6x6x6x6x6x6x6 = $6^8$ = 1,679,616 combinations.

One additional permutation is required for rotation of columns into rows producing a factor of 2.

The total combinations are 2 x 9! x $6^8$ = 1,218,998,108,160 ~ 1.2 trillion puzzles from a single pattern.

Wikipedia reports that there are 5,472,730,538 unique puzzles. We need to multiply this number of unique puzzles by 1,218,998,108,160 which we derived above to produce a grand total of 6,671,248,172,291,458,990,080 ~ 6.67 Septillion puzzles.

Wikipedia reports that there are only 6,670,903,752,021,072,936,960 puzzles. **Why the difference?**
Professor Frazer Jarvis, University of Sheffield, who wrote the original mathematical exposition for Wikipedia points out that the while the illustration above is accurate in most cases, it does result in a slight over counting because a small number of the permutations are redundant because they mirror each other.

## *Origins of Sudoku*

数独 [Sudoku in Japanese]

The name **Sudoku** or more correctly 数独 comes from Japan and consists of the Japanese characters **Su** (meaning 'number') and **Doku** (meaning 'single' or "unmarried") but the puzzle itself was invented in America based on an idea from Switzerland, that then became popular in Britain, thanks to a New Zealander who found it in Hong Kong. The first recognizable Sudoku puzzle appeared in May 1979 issue of Pencil Puzzles & Word Games by Dell magazine under the name of Number Place. Its inventor is generally agreed to be Howard Garns, a retired Indianapolis architect who died in 1989 before the Sudoku craze emerged. The puzzle was renamed Su ("number") Doku ("single") in Japan where it became very popular. **The complete Japanese title for Sudoku is "suji wa dokushin ni kagiru" which translates to: "Numbers must remain single."** The book "Mastering Sudoku Week by Week, " Paul Stephens, contains a more interesting narrative of the origins of Sudoku.

Sudoku has its deep roots in ancient number puzzles and later in the Latin Cell devised by the renowned Swiss mathematician Leonhard Euler. For many centuries people have been interested in creating and solving puzzles. Latin Cells were a building block in the development of important mathematics. An illustration of how to produce a Latin Cell and a Super Latin Cell will be provided at the end of this paper, or at the website www.brophyblog.net/latinsquares.

**Sudoku Terminology**: First let's cover again some of the conventions. The entire puzzle area is called the **grid**, it is divided into **rows** (9 horizontal cells) and **columns** (9 vertical cells) and nine **Boxes** each containing 9 cells. **Cells** are sometimes called **Squares**. We will stick with Cells, memory permitting. A **row**, **column** or **box** is called a **Group**. The **Grid** contains **27 Groups**. **A Sudoku Unit** **of 21 cells contains a row, column and box that intersect at a unique cell.** **This is an important concept; must be mastered!**

## Identification of Cells:

It would seem that the preferred method of identification is **ROW, COLUMN** in that order. I.e.: **(row, column)**

Some authors use numbers, other use letters (caps and lower case), and still others use a combination of numbers and letters.

All systems are illustrated in the Grid below. **The recommended system is (r,c) where r = row number and c = column numbers. So (5,6) = cell in the fifth row, and sixth column.**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| R1C1 | R1C2 | r1c3 | r1c4 | 15 | 16 | (1,7) | (1,8) | (1,9) |
| R2C1 | r2c2 | | | | | | (2,8) | (2,9) |
| r3c1 | | | | | | | | (3,9) |
| 41 | | | R4C4 | | r4c6 | | | 4i |
| 5a | | | | (5,5) | | | | Ei |
| 6a | | | 6d | | Ff | | | rFci |
| Ga | | | | | | | | rGci |
| Ha | Hb | | | | | | | RHCi |
| Ia | Ib | Ic | Id | Ie | rIcg | rIcg | RICh | RICi |

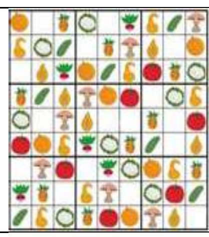
## Identification of Boxes:

The Boxes can be numbered 1,2,3,4,5,6,7,8,9 as indicated below.

Or the Boxes can be identified with two numbers representing row and column.

For example, the fourth Box (i.e. Box 4) in the illustration below could also be numbered (2,1) indicating second row, left column. The author uses the (row,column) method

| | | |
|---|---|---|
| 1 (1,1) | 2 (1,2) | 3 (1,3) |
| 4 (2,1) | 5 (2,2) | 6 (2,3) |
| 7 (3,1) | 8 (3,2) | 9 (3,3) |



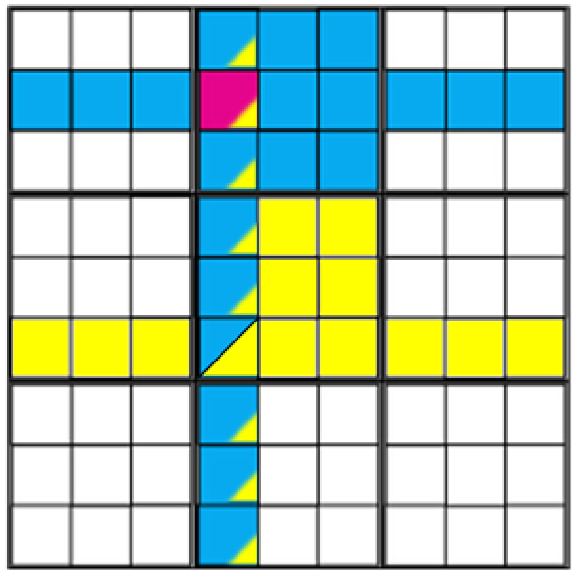



## The Sudoku Unit {very important}

Every cell in Sudoku is connected to 20 other cells. The value of a given cell (red in the example) cannot be duplicated in any of the other 20 blue cells. See **Subitizing on page 49.**

**The Advanced Sudoku player must also develop an intuitive awareness of the Exclusionary information conveyed by the Blue/Yellow cell when it is empty within the Blue Unit.**

**For example, the Blue/Yellow cell also belongs to the Yellow Sudoku Unit which shares some of the blue cells in the 2 Common Groups. Although the blue/yellow cell is empty, it imposes an Exclusionary Force on the Red Cell because the blue/yellow cell is also constrained by the values in the Yellow Cells.**



The blue cells within the Unit are sometimes referred to as the **Buddy Cells** of the **Red Cell**.


## Scanning or Eyeballing:

When you find yourself getting up tight during the course of the game; take a deep breath and just relax and hope for an insight. Always scan during the initial review of the puzzle (or anytime for that matter) to identify low hanging fruit. Most people with limited knowledge of the various strategies outlined in this paper still can identify the correct candidate (or value) for a few cells using sheer insight. The brain is amazing. **Of course, you will need a strategy and methodology to be an efficient and successful Sudoku Player** with more difficult puzzles.

## How to Play Sudoku:

The process of solving a Sudoku Puzzle is to fill in all the empty cells. However each cell has only one solution as it must obey the one Simple Sudoku rule mentioned on **page 5 and repeated here:** there is but just **one simple rule** controlling where you can place numbers or symbols in the grid. If you stare at a 9x9 Sudoku Grid, you will observe that there are **9 Columns, 9 Rows and 9 Boxes** (also called Regions or Blocks). **The simple rule is that a symbol must occur once and only once in each of the 27 Groups of nine cells**.


Let's review the **Rules for Playing Sudoku** in more detail. These rules were obtained from ***The Mensa Guide to Solving Sudoku by Peter Gordon and Frank Longo***. Surprisingly, no mention of these rules can be found in the article on Sudoku in Wikipedia: http://en.wikipedia.org/wiki/Sudoku.

**1.** **There is one simple rule in Sudoku: each Sudoku Group of nine cells must have a unique occurrence of each of the numbers 1 to 9. (Or whatever symbol set one chooses to use)**

**2.** **The Puzzle cannot contain more than 30 completed cells at the Start. As an aside, it appears (but not proven) that a minimum of 17 cells is required to produce a puzzle with a unique solution.**

**3.** **The cells that are occupied with numbers at the outset of the game have to be symmetrical. It seems that this rule is ignored many times.**

**4.** **Another is that the puzzle must be solved with pure logic. This rule is often ignored. If you are forced to guess, then check the answer sheet to make absolutely sure that your guess is correct. Rather than guess why not access one of the Sudoku on-line tools to provide some logical basis to support the guess.**

**5.** **The final rule is that puzzles can only have one answer. This is an important rule because certain conclusions and moves can be made based on this rule. More examples on this rule later in the tutorial.**

**Tips for Efficient Playing**: Completing a Sudoku Puzzle is a source of great satisfaction. Discovering an inconsistency or duplicate entry three-quarters through the puzzle is disheartening since recovery is usually impossible. **The author believes that clerical errors are the primary reason why puzzles go unsolved.** Here are some suggestions

## A. Minimizing Clerical Errors:

1. The player must choose a **Markup or Notational System:** to keep track of the potential values in each/every empty cell. The source of most clerical errors: (1) A potential value [or legal candidate at a given point in time] was **erroneously not entered** in the appropriate cell; or, (2) an impossible (illegal candidate) value was erroneously entered in the cell; candidate; or, (3) a potentially legal value is erased by error, or an illegal value or candidate that should have been removed was not erased. Clerical errors eventually lead to incorrect choices that seem so very logical at the instant, until much later when they produce an inconsistency. A few of the various administrative markup systems are discussed under the caption **"Markup Methods." [Please note that we will use the word value and candidate interchangeably.]**

2. The player must decide on the **venue for the game**. Should the player use the original paper from the newspaper or puzzle book which usually doesn't contain sufficient space for **readable Markup Notations**, or should the player copy the puzzle to a larger roomier puzzle diagram? This author transfers his puzzles to an 8.5x11" sized paper which contains two 9x9 grids which were drawn with pencil and ruler. Of course, the author makes a pile of copies of the 9x9 grids for future use. In most instances, the author can solve two puzzles on a page; but on difficult puzzles, the author uses the second grid for notes.

3. The player must decide whether and when **to "peek at" or "check" entries**. The author believes that one should never "peek" for an answer so as to recover from a stall. This practice usually destroys the rhythm of the game and is futile because it only leads to more peeks and guesses. But the author does recommend that players periodically check their entries and correct mistakes immediately. Again, it is OK to check your entry, but do not use the knowledge gained about other cells unless and only if one can prove the entry through logic.

4. **OK then! What should one do when totally bogged down and stalled?** Put the puzzle down and come back to it later. We will discuss this later in the tutorial.

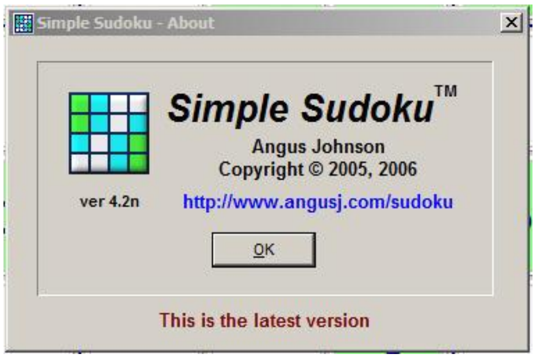
5. The player must decide whether and **how to use computerized Sudoku tools**, assuming some degree of computer and internet literacy. The author avoids using computerized Sudoku tools except for study and learning. The author enjoys entering new difficult puzzles into a **Sudoku Solver** to obtain a logical step by step solution path. The step by step path is very useful when working on very difficult puzzles to discover and learn the use of advanced logical thinking. Sudoku Solvers and internet sites will be discussed and illustrated later in this paper.

6. Do not guess. Set the puzzle down and come back to it later. Start a new easier puzzle.

## B. Markup Methods:

The illustration below was produced by a program called **Simple Sudoku**. It is free and can be downloaded to your computer from the website shown to the right. It is very simple to use, providing a selection of easy to very difficult puzzles.

Simple Sudoku is an excellent learning tool, since it steps one through the solution with hints, thereby providing a good learning experience. Unfortunately, the program does not run on the Apple products because of their proprietary architecture.

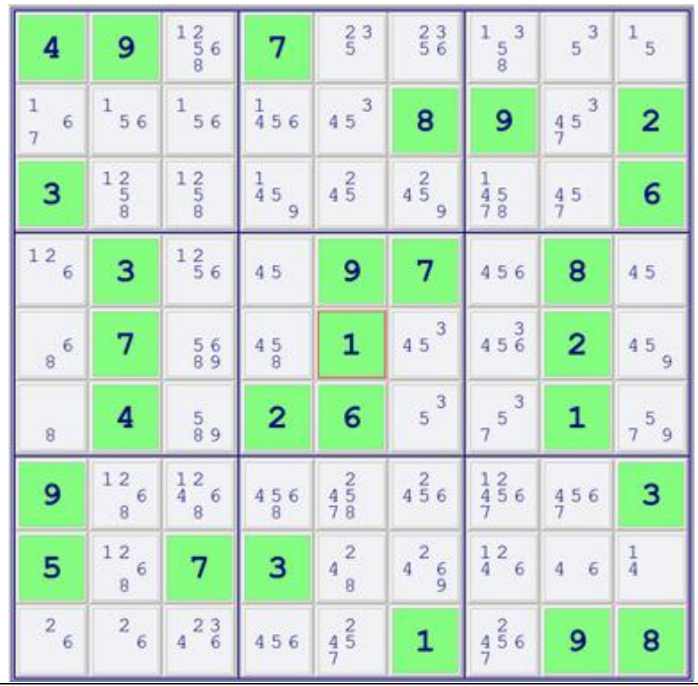


**DETAILING**: The puzzle on the right came with 27 original entries highlighted in green. The cell was then marked up with all legally possible values or candidates in the empty cells. The number of potentially legal entries is more than 200.

If this puzzle had only 18 original entries, then the cell might possibly be filled with more than 300 to 400 potential entries.

If one is patient and very accurate, this approach might be worth the investment in time since it will greatly facilitate spotting all the patterns that are described later in this tutorial.

But if errors of omission or commission are made, then the chances of completing the puzzle are greatly diminished!



**MUDDLING:** Another sensible approach is called **Muddling through the puzzle**. The Brain is an amazing tool. Somehow our brains intuitively discover solutions to Sudoku puzzles which are why Sudoku is so popular.

Scan the puzzle for low hanging fruit – an intuitive approach. Your subconscious will find some solutions.

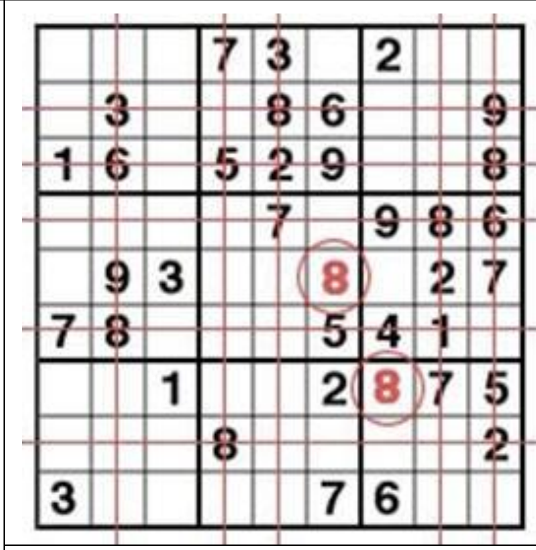
Use the simple intuitive tool of **cross hatching to force values**. Pick a high frequency usage number like 8 in the illustration. Draw lines through all the rows and columns with an 8 in it. Voila! Solutions appear from nowhere.

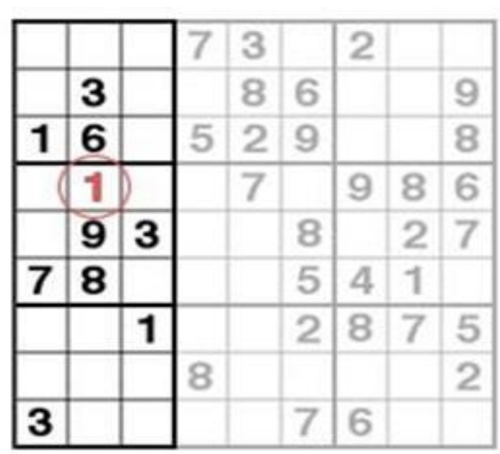
Focus on Groups (i.e. Rows, Columns, Boxes) with only 2 or 3 empty cells. The illustration on the bottom right produces a solution for Box (2,3) through a logical force: the only possible answer.

The illustration on the immediate right shows a common **force of a 1 in cell (4,2)**, another example of **cross hatching**.

Can you logically determine the candidates in the two empty cells in Box (1,2)? [cells r1c6 and r2c4 to the right]

Remember to work the **Sudoku Unit** (illustrated on page 9) when you study every cell; make in a mental drill and within a short time it will become intuitive. A Sudoku Unit 21 cells, from an intersecting row, column and box.)



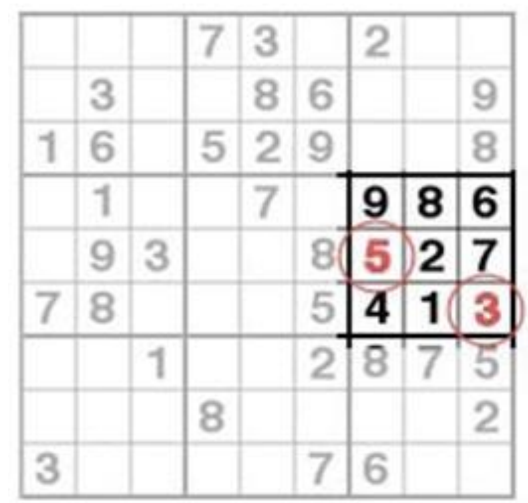


**W**rite in potential value candidates in empty cells in clusters where only 2 or 3 entries are required.

**On many puzzles, the Muddling approach can carry the day**. On more difficult puzzles, the proverbial well will eventually run dry, and one will be forced to **Detail** every empty cell.

When doing so, make sure to double check cells in which you already had some inserted partial entries.

{The Christian Science Monitor Website contains clearly written strategies for Sudoku. Try it at the internet site: http://www.csmonitor.com/homeforum/sudoku.html }



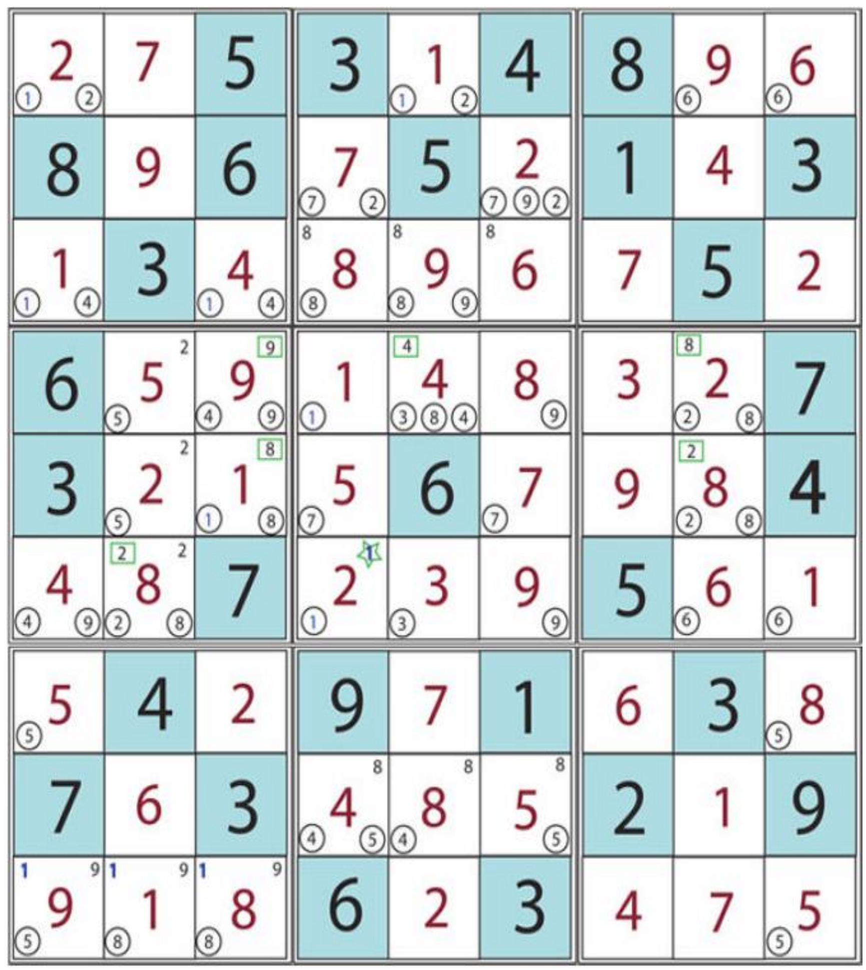
**THE PARSIMONIOUS FORCE:** A markup system and a disciplined and deliberate method of working through the puzzle from the top left corner to the bottom right corner, examining **each completed cell and each empty cell** to ascertain whether the completed cell can force an entry somewhere in the 9x9 grid; or whether the empty cell under scrutiny is in fact a **naked singleton.**

It takes repetitive eye training **(herein called subitizing – page 50)** and many hours of practice to learn to focus and to read empty cells by focusing on the contents of its buddy cells in its respective **"Sudoku Unit;"** see page 9.

**The goal of the Parsimonious Force** to delay as long as possible and eventually, to completely (with practice) eliminate the need to identify the potential values (candidates) in each square or cell on more difficult puzzles.

On very difficult puzzles, laced with X and XY and Y-Wing logic kernels, it may be wishful thinking to entirely avoid identification of any detailed potential candidates. but, they certainly can be delayed. The method improves speed and accuracy.

**Under this method, Potential Forces are called circled pairs or triplets** – see the illustration to the right above. They make a **Claim** on the territory in a Group (column, row or box.) No other



This method was developed by the author in 2004 and it has greatly improved his speed and accuracy.

A similar idea of "**minimal** entries" is used in a book published in 2007 which seems to be already out of print: ***"Top Secret Sudoku System"*** by **Stephen Kerr**. Also, the author, Peter Gordon, of ***Mensa Guide to Solving Sudoku*** also claims to have devised a forcing approach using limited entries, but he does not profess to use the method.

| | |
|---|---|
| entries are allowed.<br>**With this Force Method the puzzle either solves or stalls.** When the stall is resolved, the method is continued. | **Neither of these approaches develops the concept of reading empty cells, which requires an understanding of the Sudoku Unit, see page 9, and the eye training required to perfect subitizing. [Research subitizing for children!]** |
| The **Parsimonious Force** is illustrated in great detail between the **following 2 RED TABS**. It is worth a peek! Note how neat and orderly the solved puzzle looks. Note how few entries had to be made. Note that none of the potential values that were entered had to be erased, thus providing an audit trail. It provides a disciplined systematic method and plodding through every cell, step by step, looking in both horizontal and vertical directions. **Worth a study**. | |

# The Parsimonious Force:

## *A Fast Algorithm to Solve Difficult Sudoku Puzzles*

***Quick Start - Rules:*** For the **serious Sudoku player**: here is a useful approach when you want to develop the skills to work on very hard puzzles and to avoid or to minimize detailing all the potential entries for each cell. At the beginning of a puzzle, the detailing **effort could require as many as 200 to 500 entries** and is fraught with clerical errors. The proposed method described herein is designed to improve your speed, accuracy and completion rate. It requires analyzing cells with entries as well as empty cells (**or "reading empty cells**.") Since it takes eye training to analyze empty cells; the initial emphasis in this immediate exercise is simply on **"GETTING STARTED"** with cells with entries.

This method either **Solves or Stalls**. If the puzzles **Stalls**, then use the concept of the **Sudoku Unit** (see page 9) on one of the more **populated Groups** to ferret out **Exclusionary Forces**. When the Stall is resolved, continue with the Method.

If you are venturesome or a quick study, go directly to the puzzle solution below and follow the step by step solution of the puzzle. It is important that you intuitively grasp how and why forces, both **positive and exclusionary forces**, occur. If you become bewildered, come back and read the quick start rules below. **A suggestion: get an 8.5 x 11 sheet of paper, and draw two 9x9 Sudoku cells on it.** Then make copies of the drawing for future use. Transfer your Puzzle onto the sheet so that you have more room to make notes. The page 49 of this booklet book contains a copy of two blank 9x9 grids on an 8.5x11 inch sheet of paper.

1. **Unfortunately, this method is not rocket science - just a grind out the sausage discipline.**
2. **For this exercise, do not use your own personal markup system, please. Do not rush to clutter the page with a lot of potential values.**
3. **Let this method find the pertinent values to markup.**
4. **Do not erase markups using this method; they provide an audit trail.**
5. **Do a quick Mental Scan for Low Hanging Fruit: "the easy pickin's."**
6. **Try your favorite Mental Cross-hatching to discover more Singletons.**
7. **Enter these values, but do not markup the page with your personal method, please.**
8. **Always be alert for EXCLUSIONARY FORCES that work on empty cells; it must become an intuitive part of your scan process. Please review the graph on 9 which illustrates the simple features of The Sudoku Unit.**
9. **EXCLUSIONARY FORCE: When examining an empty Cell in a Sudoku Unit (previously defined on 9), note that it has 20 Buddy Cells, and each Buddy Cell belongs in turn to its own Sudoku Unit. Although a Buddy Cell being scrutinized may be empty, IT CONTAINS INFORMATION; its potential values are constrained by cells in its own Sudoku Unit. A value in a cell can be forced when it is determined that other values are excluded because of exclusionary constraints in other Sudoku Units.**

10. **EXCLUSIONARY FORCE: Said another way. (1) What values (candidates) can exist in the cell under consideration? (2) what naked or hidden singleton value MUST exist in the cell, because OTHER VALUES ARE EXCLUDED by constraints in other "empty buddy cells" in the Sudoku Unit. Yes, this can get to be a recursive exercise but you can with practice become an eagle eye. otherwise, you might want to be content to work on easy and moderate puzzles. Remember, the values in these "empty cells" are constrained by the intersection of the respective Sudoku Units they belong too. And, do not forget the Boxes, and Alleys in the boxes. We should define an** alley **somewhere in these pages.**
11. **START: Now start in Cell (1,1); work left to right and down the page to cell (9,9).**
    **ALTERNATIVE START – IN SEQUENCE. another way: start with the numeral 1 and work all the 1s, left to right, and down the page. Then start with the 2s, etc.**
    **ALTERNATIVE START – BY HIGH FREQUENCY. another way: and preferred by the author, start with the numeral that appears with the highest frequency, and so forth.**
12. **REMEMBER: Cell (1,1) means Row 1, Column 1; Box [2,3] is the Box in box-row 2, box-column 3.**
13. **If the cell is blank, read its Sudoku Unit (ignore pro temp); move right to the next cell.**
14. **If the cell contains a Given or Solved VALUE, study the VALUE.**
15. **Let's call the cell being studied, the WORKING CELL.**
16. **Examine the 2 other contiguous horizontal Groups (rows) it can impact. (see cross hatching)**
17. **Examine the 2 other contiguous vertical Groups (columns) it can impact. (see cross hatching)**
18. **What are we looking for? Why are we doing this? We are attempting to keep Markups to an absolute MINIMUM to reduce clerical errors. [DIFFICULT puzzles are DIFFICULT to complete because we make clerical errors.]**
19. **Remember this, please! We're only looking for 3 TYPES OF POTENTIAL ENTRIES: Singletons, Doubletons [Circled Pairs] and Tripletons.**
20. ***FIRSTLY*, look for a positive or exclusionary force of a unique VALUE (a Singleton) in another cell. Usually these are picked up in the low hanging fruit scan. But they keep appearing as the solution of the puzzle progresses.**
21. ***SECONDLY*, look for the potential placement of a pair of identical potential VALUES in two cells in a Sudoku Unit. They lay Claim to territory in a Group (row, column, box).**
22. **These pairs are called Circled Pairs (or Doubletons); they become part of a logic chain which will be discussed later. Take a look at the illustration immediately [page 12] above and note the circled pairs. Compare that grid to the grid associated with the Detailed Markup System Illustration, also above on page 11. The Circled Pair routine has far fewer entries and conveys more information with eye scans.**
23. **Circled Pairs help identify the so-called Buzz Words (naked pairs & triples, X-wings, etc). Place the Circled Pairs in the lower left or right corner of the appropriate cells**
24. **Do not be tempted to markup, if a VALUE can exist in 3 or more cells. Please, IGNORE IT for the time being. If the Parsimonious Force method hopelessly stalls, you will have ample opportunity to Markup to your heart's desire.**
25. ***THIRDLY*, look for the placement of a TRIPLET in a Group. An aligned triplet is 3 identical VALUES in an Alley in a row or column in a Box. Place the triplet VALUES in an upper right or left corner of each cell without circles. Triplets are important; they lay Claim to a territory; they enable forces in other columns, rows and Boxes. Remember, a tripleton can only appear in an Alley.**
26. **When you find a solution for a cell or cells, whether it be Singleton force, Circled Pair or Triplet, study its Sudoku Unit and** follow the **solution trail** that might be enabled**. Meaning--determine whether the new entry or entries enable new forces in its Sudoku Unit pathway.**
27. **When you have exhausted the CURRENT WORKING CELL, move to the next working cell, and continue until you reach the last given or solved cell or cell (9,9) in Box [3,3].**

**28.** **At this juncture you have several options. The author prefers to start back at Cell (1,1) and work through the puzzle again. Of course, it will go much faster the second time through. If the second run-through goes very quickly, then you are approaching a Stall. Other techniques will be required.**

**29.** **The worst case scenario at this juncture, is making the decision enumerate in DETAIL all the remaining empty cells. That task, however, should be less burdensome, since many of the original empty cells have been solved. However, the author prefers to focus on Groups with only two or three empty cells, and to work the EXCLUSIONARY forces associated with their respective Sudoku Units. In other words, get serious about reading the empty cells in high frequency Groups.**

**30.** **Rather than beat this subject to death, try Puzzle #1. Puzzle #1 was rated Beware, Very Challenging by *Bill Shortz*, puzzle editor for the NYTimes.**

**31.** **If you want a tougher assignment, try Puzzle #3, labeled Diabolical (attached below)**

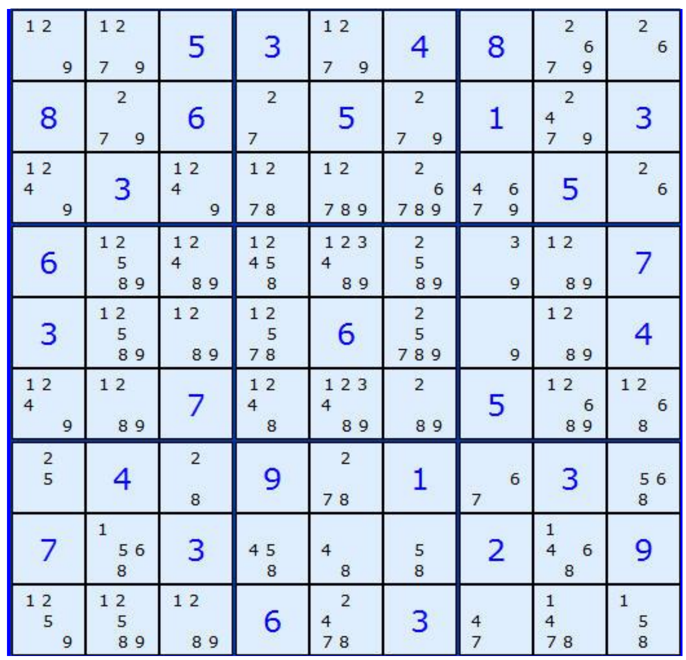
**32.** **Puzzle #3 runs into a STALL because it is bifurcated, and a GUESS must be made. The requirement to GUESS is BAD, and should not be allowed, but puzzle makers make them, perhaps unwittingly.**

**33.** **When you run into a STALL early in the puzzle, You may have missed CLUES OR it's a bad puzzle. Do a quick scan of the audit trail with emphasis on Exclusionary Forces. Otherwise discard the puzzle. STALLS near the end are OK since they actually add interest. Do not panic**

**34.** **Puzzles should be solvable with pure logic, not guesses. But some of the really difficult puzzles are bifurcated; if trifurcated then discard the puzzle.**

**Puzzle #1: Rated BEWARE by Bill Shortz, Sudoku Editor, NYTimes**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 5 | 3 | | 4 | 8 | | |
| 8 | | 6 | | 5 | | 1 | | 3 |
| | 3 | | | | | | 5 | |
| 6 | | | | | | | | 7 |
| 3 | | | | 6 | | | | 4 |
| | | 7 | | | | 5 | | |
| | 4 | | 9 | | 1 | | 3 | |
| 7 | | 3 | | | | 2 | | 9 |
| | | | 6 | | 3 | | | |



**Do not use the Detail Markup shown above right; it is shown for illustrative purposes. It was produced using Angus Johnston's Simple Sudoku program, which is free, and which can be downloaded from www.angusj.com. Unfortunately, it does not run on the Mac. It is shown here for illustrative purposes, to highlight the amount of effort that is required to map out all the potential candidates (values.) Once an effort is made, (assuming it is done accurately), the step by step solution of difficult puzzles is greatly simplified; it is an option for puzzle solvers, but not preferred by the author. The Parsimonious Force technique is designed to arrive at a correct solution without all the Detailing effort required to produce the above Map. Of course, if one uses Simple Sudoku the effort is also greatly diminished. Simple Sudoku is a great interactive learning tool. But it is also cumbersome. Anyway, we ought to use the computer to take the drudgery out of pre-filling all the potential candidates when we are building our skills to solve DIFFICULT puzzles. It will provide a learning opportunity in identifying the more complicated patterns like XY-Wings and Chains. The**

**advantage of the Parsimonious Force technique is that is allows us to develop the mental acuity, with practice, to "read" empty cells and identify naked and hidden singletons.**

**STEP BY STEP SOLUTION TO THE PUZZLE: (this illustration ignores empty cells pro-temp.)**

**1. Starting with 5 in Cell (1,3), a pair of ((5))s is required in cells (4,2) & (5,2); as well as (7,1)& (9,1)**
**2. Starting with 3 in Cell (1,4), a pair of ((3))s is required in cells (4,5) and (6,5) [do not jump ahead.]**
**3. Starting with 4 in Cell (1,6), a 4 is forced in Cell (2,8) because of (7,2), and a pair of ((4))s in Cells (3,1) and (3,3); and a 4 is forced in Cell (9,7); and a pair of ((4))s is forced in Cells (8,4) and (8,5); and a pair of ((4))s in Cells (4,3) and (6,1)**
**4. Starting with 8 in Cell (1,7), an 8 triplet in Cells (3,4),(3,5) & (3,6)**
**5. Starting with 8 in Cell (2,1), no additional information.**
**6. Starting with 6 in Cell (2,3), a 6 is forced in Cell (3,6), and a pair of ((6))s in Cells (1,8) and (1,9); and a pair of ((8))s in Cells (3,4) and (3,5); and a 6 is forced in Cell (8,2)**
**7. Starting with 5 in Cell (2,5), a pair of ((5))s is required in Cells (8,4) and (8,6)**
**8. Starting with 1,4 in Cell (2,7) and Cell (2,8); no additional information**
**9. Starting with 3 in Cell (2,9), a 3 is forced in Cell (4,7) and (6,5). Starting with 3 in Cell (3,2), pass.**

*NOTE: the 3s have been completely solved. See the suggestion about keeping track of Values that have been completed Marked Up and Values that have been completly solved: See tips below, page 17 at ***HELPFUL HINT.*

**12. Starting with 6 in Cell (3,6), a pair of ((6))s is required (by scanning ahead) in Cells (6,8) and (6,9) which produces an X-wing with [1,3]; which forces a 6 in Cell (7,7) which forces a 9 in Cell (5,7) (exclusionary force); which forces a 7 in Cell (3,7) (column 7 force); which forces a 7 in Cell (9,8); which forces a 7 in Cell (7,5) which requires a pair of ((7))s in Cells (2,4),(2,6), (5,4) and (5,6) which produces an X-wing; which forces a 7 in Cell(1,2); and forces a 9 in Cell (1,8); and forces a 6 in Cell (1,9); and forces a 6 in Cell (6,8); and requires a pair of ((9))s in Cells (2,6) and (3,5) and forces a 2 in Cell (3,9); which requires a pair of ((2))s in Cells (4,8) and (5,8) and forces a naked pair ((1,2))s in Cells (1,1) and (1,5) [let's take a breath.]**

*NOTE: the 6s have been completely solved; and all 7s accounted for.*

**13. Starting with 7 in Cell (3,7), pass**
**14. Starting with 5 in Cell (3,8), a pair of ((5))s is required in Cells (7,9) and (9,9) which produces an X-wing with [3,1]**

**15. Starting with 2 in Cell (3,9), no additional information**
**16. Starting with 6,3,7 in Cells (4,1), (4,7), and (4,9), pass**
**19. Starting with 3,6,9,4 from Row 5, pass & no additional information**
**20. Starting with 7,3,5,6 from Row 6, pass, no additional information; 21. Starting with 4 in Cell (7,2), pass**

**22. Starting with 9 in Cell (7,4), a 9 triplet is required in Cells (9,1),(9,2) and (9,3)**
**23. Starting with 1 cell (7,6), a 1 triplet in required in Cells (9,1), (9,2) and (9,3) which forces a 1 in cell (8,8) and (6,9), which forces a pair of ((8))s in Cells (4,8) and (4,9) as well as (7,9) and (9,9)**
**24. Starting with 6,3 in Row 7 and 7,6,3 in Row 8, pass**
**25. Starting with 2 in Cell (8,7), a 2 will be forced in Cell (9,5), which forces a 1 in Cell (1,5) and a 2 in Cell (1,1) and (7,3), and an 8 triplet in Cells (8,4), (8,5) and (8,6), and a pair of ((8))s in Cells (9,2) and (9,3), which forces an 8 in Cell (7,9) and a (5) in Cell (9,9) and (7,1), and a pair of ((2))s in Cells (2,4) and (2,6), which forces a 9 in Cell (3,5), and an 8 in Cell (3,4), which requires a 4,8 in Cell (4,5), and a pair of ((1))s in Cells (3,1) and (3,3), which forces a 9 in Cell (2,2)**

**26. Starting with 6,3,8 from Row 7, pass; 27. Starting with 7,6,3,2,1,9 from Row 8, pass; 28. Starting with 6,2,3,4,7,5 in Row 9, pass - no additional information**

***29. At this juncture the method seems to stalling*. ..but we are almost done. In steps 30 thru 34 we will use a TRUE-FALSE chain which produces an inconsistency as shown in step 34, leading to a solution.**

**30. Let's see: box [1,3] and [3,3] are solved. Box [3,1] and [3,2] contain naked triples. Box [1,1], [1,2] and [2,3] contain naked pairs, leaving only box [2,1] and [2,2] for review. Looking at the distribution of the**

unresolved ((8))s, it seems intuitively obvious that cell (6,2) must be an 8 which breaks the puzzle wide open. But let's be contrarian.

**31. A quick scan of [2,1] and [2,2] shows that:**

**Cell (4,5) contains an ((8,4)) which produces a naked pair with [3,2]**

**Cell (5,3) contains an ((1,8)) and Cell (6,2) contains a ((2,8))**

**32. To regenerate the method, we must make an educated guess!**

**Since 8 is not solved in [2,1], [2,2] and [2,3] it becomes the likely candidate**

**33. Let us assume 8 is the correct value in Cell (5,3) marked by a green square in the upper right hand corner. A chain is created as follows:**

**Cell (6,2) = [[2]] GREEN BOX; Cell (5,8) = [[2]] GREEN BOX**

**Cell (4,8) = [[8]] GREEN BOX; Cell (5,5) = [[4]] GREEN BOX**

**Cell (6,4) = <<1>> GREEN STAR, which is A CONTRADICTION with Cell (6,9)**

**34. Therefore, Cell (5,3) = 1 and Cell (6,2) = 8 {as we intuitively guessed earlier in step 30}**

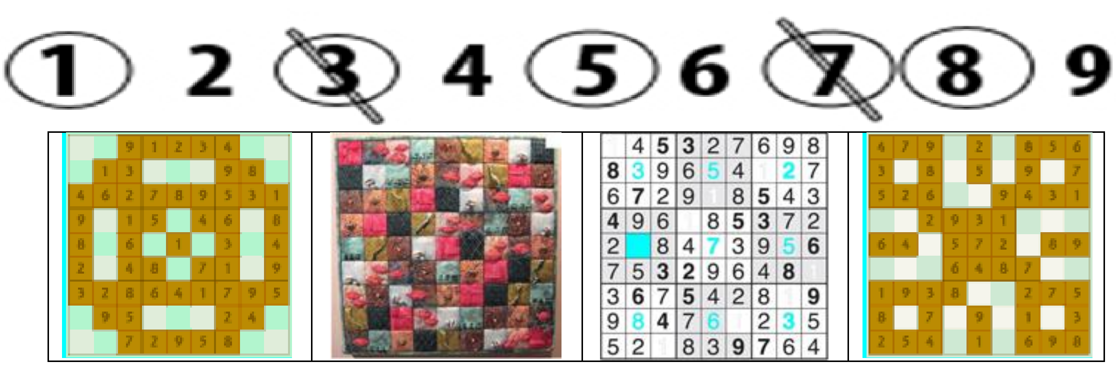
**35. An alternative elegant move that breaks the logjam: note that Cell (9,1) contains candidates 1,9; Cell (9,2) contains 1,8 and Cell (9,3) contains 1,8,9. If Cell (9,2) is an 8, then both Cells (9,1) & (9,3) would contain the candidates 1,9. This is illegal since the puzzle would not have a unique solution. Cell (9,2) must equal 1.**

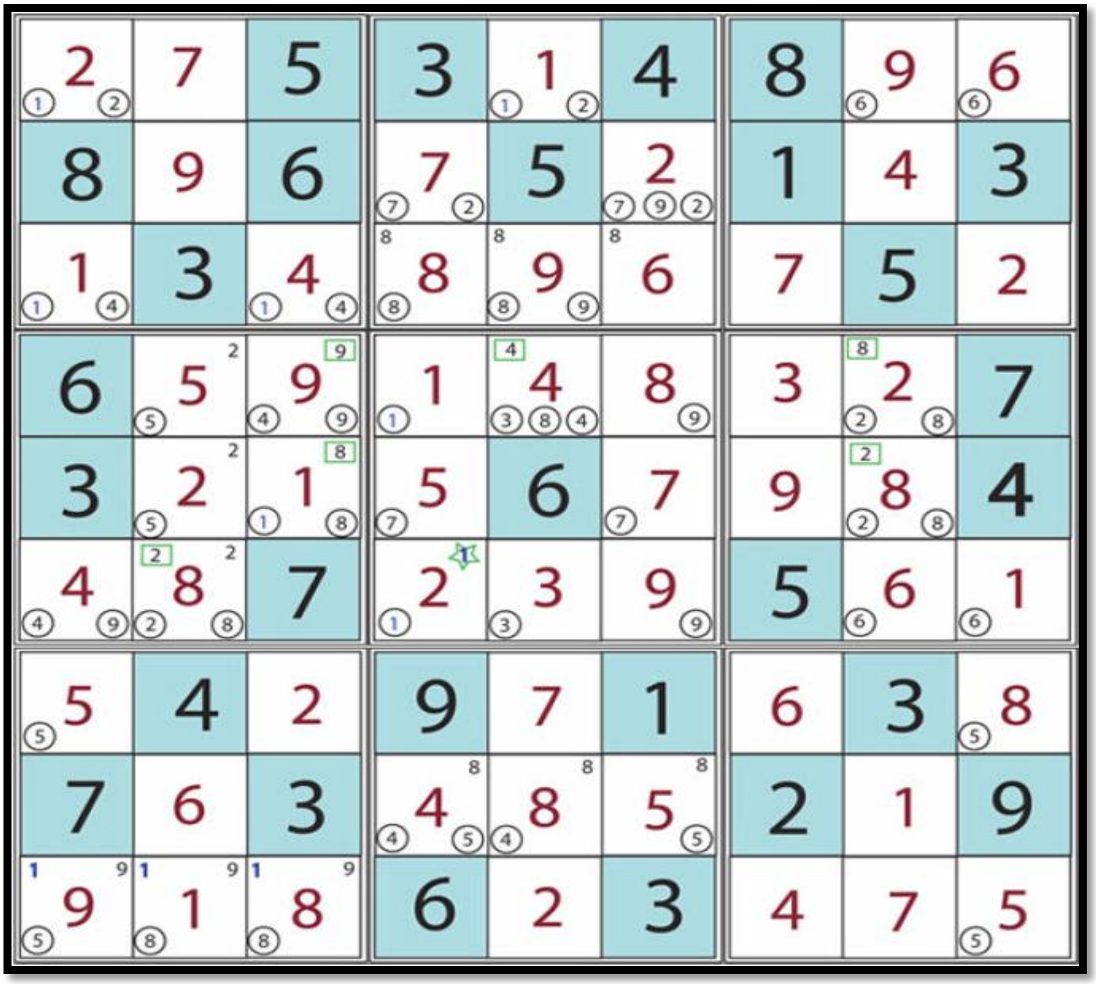
**36. Therefore Finish and Mop-up:**

**Cell (9,1) = 9; Cell (9,3) = 8; Cell (4,3) = 9; Cell (6,6) = 9; Cell (6,1) = 4;**
**Cell (3,1) = 1; Cell (3,3) = 4; Cell (5,3) = 1; Cell (4,4) = 1; Cell (4,2) = 2,8;**
**Cell (5,2) = 2,8; Cell (6,2) = ; Cell (6,4) = 2 – only choice; Cell (6,2) = 8;**
**Cell (2,4) = 7; Cell (2,6) = 2; Cell (5,4) = 5; Cell (8,4) = 4; Cell (8,5) =8;**
**Cell (4,5) = 4; Cell (4,6) = 8; Cell (8,6) = 5; Cell (4,8) = 2; Cell (5,8) = 8;**
**Cell (5,6) = 7; Cell (4,2) = 5; Cell (5,2) = 2. QED**

*****HELPFUL HINT:** It is essential to keep track of all the potential values or candidates in play. Start by writing the numerals 1, 2, 3, 4, 5, 6, 7, 8, 9 across the top of the page of the puzzle. Then, when you discover or determine that you have identified all the 6's, {as an example,} in the **Parsimonious Markup**, then place a circle (the example looks more like ellipses) around the 6. Subsequently, when one discovers or ascertains that all the 6's have been solved, then place a slash through the circled 6. This is an easy way to minimize your searches for new potential candidates.

**KEEPING TRACK OF CANDIDATES**: In the following example, the notation implies that all the 3s and 7s have been solved; and that the markup process has identified all of the 1s, 5s, and 8s.





**STEP BY STEP SOLUTION TO THE PUZZLE:** **(this procedure focuses on empty cells pro-temp.)**

1. **Assume that we use our favorite scanning techniques to capture low hangin' fruit.**
2. **Assume also that we have, at least at a cursory lever, analyzed the given entries for structure and potential circled pairs and tripletons.**
3. **a naked 9 found in cell(5,7); a naked 3 found in cell(4,7); a naked 6 found in cell(3,6); a naked 2 found in cell(3,9); a naked 6 found in cell(1,9); a naked 3 found in cell(6,5);**

4. **INTERRUPT: It should be patently obvious (after the fact) why each of the aforementioned values is a naked singleton. The issue now is how to develop the mental and visual acuity to identify these singletons in the normal course of solving the puzzle. One needs to subitize each empty cell within the constraints of its respective Sudoku Unit.**

5. **a naked 6 found in cell(6,8); a naked 6 found in cell(8,2); a naked 6 found in cell(7,7); a naked 4 found in cell(2,8); a naked 7 found in cell(3,7); a naked 9 found in cell(1,8); a naked 4 found in cell(9,7); a naked 7 found in cell(9,8); a naked 7 found in cell(7,5); a naked 2 found in cell(9,5); a naked 1 found in cell(1,5); a naked 2 found in cell(1,1); a naked 7 found in cell(1,2); a naked 9 found in cell(2,2); a naked 8 found in cell(3,4); a naked 9 found in cell(3,5); a naked 5 found in cell(7,1); a naked 8 found in cell(7,9); a naked 1 found in cell(6,9); a naked 2 found in cell(7,3); a naked 1 found in cell(8,8); a naked 5 found in cell(9,9);**

**6. Note that at this juncture, the puzzle is beginning to stall. One alternative, pursued here, is to bite the bullet and enumerate the remaining candidates, since it offers several learning opportunities. First, we will view the status of the puzzle, after detailing the remaining potential candidates.**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 | 7 | 5 | 3 | 1 | 4 | 8 | 9 | 6 |
| 8 | 9 | 6 | 2 7 | 5 | 2 7 | 1 | 4 | 3 |
| 1 4 | 3 | 1 4 | 8 | 9 | 6 | 7 | 5 | 2 |
| 6 | 1 2 5 8 X | 1 4 8 9 | 1 2 4 5 | 4 8 | 2 5 8 9 | 3 | 2 8 | 7 |
| 3 | 1 2 5 8 X | 1 X 8 | 1 2 5 7 | 6 | 2 5 7 8 | 9 | 2 8 | 4 |
| 4 9 | X 2 8 | 7 | 4 2 | 3 | 2 8 9 | 5 | 6 | 1 |
| 5 | 4 | 2 | 9 | 7 | 1 | 6 | 3 | 8 |
| 7 | 6 | 3 | 4 5 | 4 8 | 5 8 | 2 | 1 | 9 |
| 1 9 | 1 8 | 1 8 9 | 6 | 2 | 3 | 4 | 7 | 5 |

**7. The first observation is that puzzle at this juncture offers several interesting chaining opportunities; but let's ignore the chaining opportunity until later and get to the crux of the logjam.**

**8. Box [2,1] contains a Naked Quad: cells (4,2), (5,2),(5,3),(6,2) shown with the violet Xs. They contain some or all of the candidates 1,2,5,8. Four4 cells can only contain 4 values, so the candidates 1,8 must be removed from cell (4,3). Seems strange? But it is real hard logic!**

**9. The removal of the candidates 1,8 from cell (4,3) sets up the red XY-Wing in Boxes [2,2] & [2,3]. An XY-Wing always involves 3 candidates in the form XY, XZ and YZ. I.e. 3 candidates taken 2 at a time and we find them in cells (4,5),(4,8),(6,4). Actually it is a closed chain. These cells can have the values 4,8,2 or 8,2,4 respectively. Whichever set is correct, it forces the elimination of the 2s in cell (4,4) and cell (4,6). There is also a geometric component to the XY-Wing. Maybe you see it; read up on it his paper.**

**10. Next, we find another blue XY-Wing in Boxes [2,1] & [2,2]. We find them in cells (4,5),(6,2),(6,4). They contain either values 4,8,2 or 8,2,4 respectively, and in either case, the correct set will eliminate the 8s in cell(4,2) & (4,6). Notice the Geometric component that makes it possible.**

**11. The elimination of the 8s in step 10 above creates a naked singleton of 8 in cell (6,2), since it is the only cell in row 6 that can allow an 8. And this breaks the puzzle to a trivial chain of naked singles as follows:**

**a naked 8 found in cell(6,2); a naked 1 found in cell(5,2); a naked 4 found in cell(3,3);**
**a naked 4 found in cell(3,1); a naked 9 found in cell(4,3); a naked 7 found in cell(7,1);**
**a naked 2 found in cell(6,4); a naked 7 found in cell(2,4); a naked 2 found in cell(2,6);**
**a naked 5 found in cell(5,4); a naked 8 found in cell(4,4); a naked 4 found in cell(4,5);**
**a naked 1 found in cell(4,4); a naked 2 found in cell(4,8); a naked 5 found in cell(4,2);**
**a naked 2 found in cell(5,2); a naked 7 found in cell(5,6); a naked 8 found in cell(5,8);**
**a naked 9 found in cell(6,6); a naked 4 found in cell(8,4); a naked 8 found in cell(8,5);**
**a naked 5 found in cell(8,6); a naked 9 found in cell(9,1); a naked 1 found in cell(9,2);**
**a naked 8 found in cell(9,3); QED**

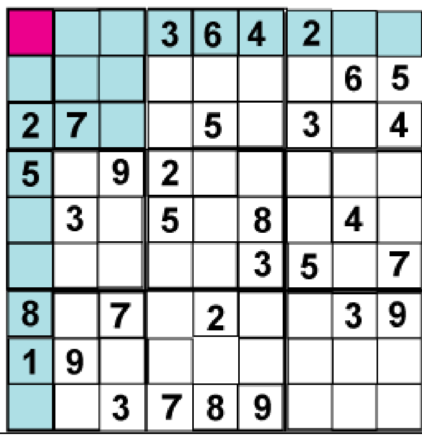
**CONCLUSIONS: If the Quads, and XY-Wings are too mind bending to absorb at this juncture; then try a TRUE-FALSE CHAIN. You can have a lot of fun with this one, and there are many paths to this chain, all of which will end with a contradiction. In the example below, we assumed that Cell(3,1) contained a 4. If the assumption of 4 is TRUE, then all the circled values are TRUE. For a while it looked as if the 4 was the correct choice but ALAS, we encounter contradictions in Cell(4,8) and (6,6). So Cell(3,1) must contain a value of 1. We should test this assumption as well before committing to it.**

# Strategies & Tools

**Naked & Hidden Candidates:** Your markup system or scanning methodology will identify a number of possible candidates in a cell. Your goal is to employ strategies to eliminate all but one of the candidates. Your analysis must focus on spotting the **Hidden Candidates** in a series of cells that will support your strategy to eliminate some of the remaining candidates. **Hidden Candidates lead to Naked Candidates**.


**Only choice rule:** There may be only one possible choice for a particular Sudoku cell. In the simplest case you have a group with eight cells completed leaving only one remaining choice available.

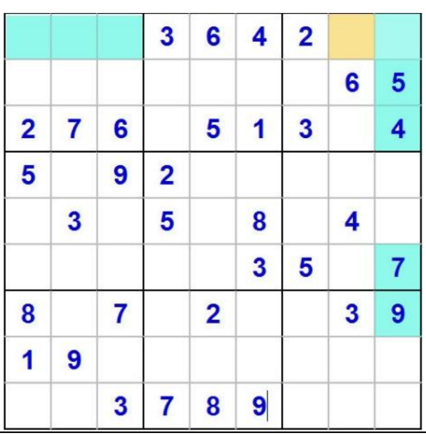
Looking at the second row all the cells except the first cell (2,1) have been allocated, so the missing number **4** has no choice but to go in the cell (2,1). You can use this technique by just scanning for 8 allocated cells in any row, column or region.




## Naked Single:
## aka The Complete:
## the Force:

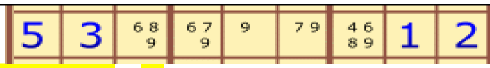
The value of the red cell must be a 9.

A Naked Single example is similar to the preceding example: **The Only Choice**. It is the only number that can fit.




**Last Cell Remaining:** The value of the orange cell must be 7.

The top row has five empty cells, and one of them must contain a 7. But it cannot go in the blue cells in the Box (1,1) since it contains a 7; nor can it go in Column 9 cell because of the 7 already in this column. It can only go in the orange cell because **it is the last cell left.**

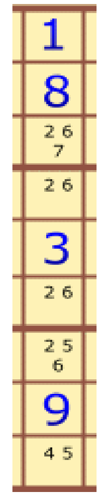


**Singles**: In the above example, you can see the naked single is the nine. All the other nines may be crossed off leaving a 6,8 pair; a 6,7 pair; a single 7; and a 4,6,8 triple. If you didn't pencil in all the possible candidates, the naked nine would be less obvious. No doubt you also noted in this example that once you solve for the naked nine, the 7,9 pair's solution became a naked single. The 7,9 pair is called a hidden single. At the right is another example of a hidden single.

## Naked Hidden Singles:

In the example at the right, there are two hidden singles. Hidden singles have only one place to reside. The extra candidates in the cell "hide" the single solution. In this example, the third cell from the top is a seven. Likewise the only number that can go in the bottom cell is a four.

When there are a lot of candidates showing from the surrounding rows, Columns, and Boxes, a hidden single can be hard to spot. Hidden singles occur often.



**Single Possibility Rule:** When you examine an individual cell you will often find there is only one possibility for the cell. If eight cells are solved in the group, it is the **only choice** rule. However, when Groups intersect you may have a Group with several unallocated cells and yet only one possibility exists for one of the cells.

In this partially solved Sudoku there are quite a few solvable cells. Look at the purple Cell (4,1): column contains: (1,2,3,4,5,8); and row 4 contains: (6,9); leaving **7** as the only possibility.

All the green cells in the grid can be solved using the single possibility rule, thereby making it a very useful rule.

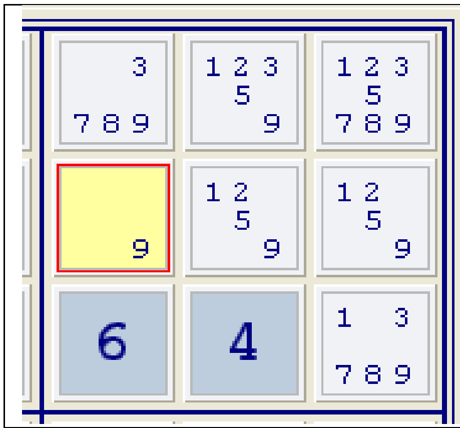
This rule is nothing more than the intuitive application of the **Sudoku Unit**, described previously on page 9. Look for Groups (row, column, box) with only 2 or 3 empty cells and solutions will appear

**Only Cell Rule:** Often you will find a Group of Sudoku cells where only one of the empty cells can take a particular number. For example, if a Group has seven cells completed and two open cells, it is often the case that the **two intersecting Groups** will impose a constraint on one or both of the two open cells. Invariably you will discover you are left with an only cell for one of the two remaining candidates.

In the instant grid to the left the highlighted column 3 has seven numbers allocated. The missing numbers are 1 and 3. But you can see that there is already a 3 in cell (9,6) so a 3 is excluded from cell (9,3); the 3 is forced to be allocated in the other cell (1,3); it is the **only cell** in column 3 where a 3 can be allocated. {to many words here, need an editor.}

You will often find that the same cell can be solved by the **'single possibility'** rule as well as the 'only cell' rule. It doesn't matter which rule you choose, it depends which one you find easiest to recognize and understand. {Where did I copy this awful text?}

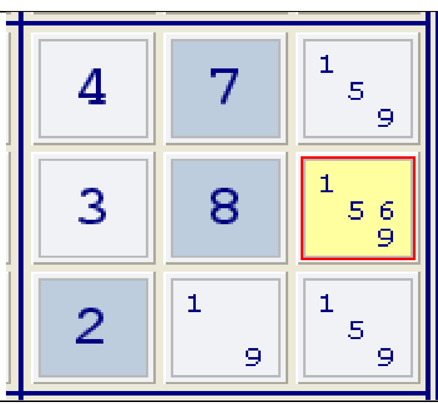
Note: Whenever there is only one cell remaining in a Group you can assign a symbol by applying either the **'only choice', 'single possibility' or 'only cell' rule** as all of them imply the same thing. A key feature of Sudoku is that it can be solved in several ways using different tactics.



**Singles:** Any cells which have only one candidate can safely be assigned that value.

It is very important whenever a value is assigned to a cell, that this value is also excluded as a candidate from all other blank cells sharing the same row, column and box.

Remember to clean up Detail Residue in other cells when you solve a particular cell.



**Hidden Singles:** Very frequently, there is only one candidate for a given row, column or box, but it is hidden among other candidates.

In the example on the right, the candidate 6 is only found in the middle right cell of the 3x3 box. Since every box must have a 6, this cell must be that 6.

## Two out of Three Rule (Slicing & Dicing):

**O**ne of the most Useful strategies involves **slicing and dicing**, but some **Sudoku authors refer to it as 'slicing and slotting'**. It is a quick way of mentally scanning and spotting cells to solve. It usually finds a cell or two that can immediately be solved. You need to work with three columnar Boxes or three row Boxes at a time. Then methodically **'crossshatch'** all the 1s, then 2s, etc. through to the 9s. Follow the following example.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 9 | 5 | 1 | | | 6 | 2 |
| 6 | 3 | 4 | | | | 5 | 9 | |
| 1 | 2 | 5 | 6 | 3 | 9 | 7 | | 4 |
| 2 | 5 | | 8 | 4 | 7 | 6 | 3 | |
| 4 | 6 | | | 5 | | | 1 | 7 |
| | 8 | 7 | 3 | 6 | 1 | | 2 | 5 |
| 5 | | 6 | 1 | 7 | 3 | 2 | 4 | 8 |
| | 1 | 2 | | | | 9 | 7 | 6 |
| 7 | 4 | | | 9 | 6 | 1 | | |

**L**ook at the top three rows where the 1s are located - they are in Cell (1,5) and Cell (3,1). There is no 1 in row 2; so it must go in Box (1,2). But there are 1s in columns 3 and 4, thereby forcing the remaining 1 in Cell (2,4). The same reasoning when applied to the next three rows shows that a 1 is forced in Cell (4,3).

**W**hen you apply this technique with the number 2, you will discover that a 2 is forced in Cell (9,4), which in turn forces a 2 in Cell (5,6) and finally a 2 in Cell (2,6).

**A**ll the rows and columns can be sliced and diced, with all the numbers 1 thru 9. Of course, you will also find that many scans will not find a solution

## Sub-Group Exclusion Rule:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | 7 | | | | 5 | 8 |
| | 5 | 6 | 2 | 1 | 8 | 7 | 9 | 3 |
| | | | | | | 1 | | |
| | | | | | | | 8 | 1 |
| | | | 3 | 7 | 6 | | | |
| 9 | 6 | | | | | | | |
| | | 5 | | | | | | |
| | | 4 | | 2 | 1 | 8 | 3 | |
| 8 | 7 | | | | 3 | | | |

**Rarely needed, but exceptionally useful is the Sub-Group Exclusion Rule. Remember that a Group is a column, row or box of 9 cells**

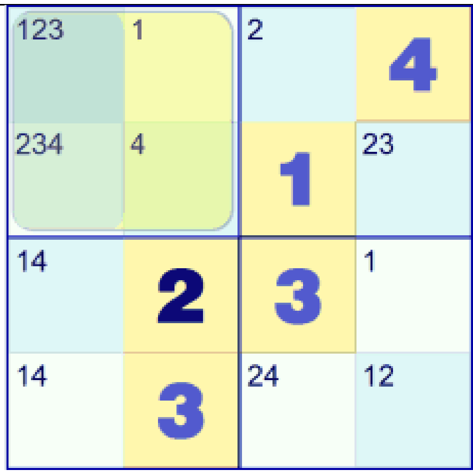
**A Sub-Group is one-third of a column or row, all in the same Box. Sometimes a sub-group is referred to as an Alley, particularly if the cells or blank. More on this later. The illustration shows 3 column subgroups in Box 1, colored lavender, saffron and pale green.**
**The exclusion rule: if a candidate is known to exist in one of the 3 cells in a subgroup, then it is excluded from the remaining 6 cells in the box.**


## Sub-Group Exclusion Rule

**(Mental Exercise):**

This puzzle comes from a 2 x 2 Sudoku Puzzle, illustrating the sub-group exclusion rule.

The sub-group in question is Cell (3,4) and Cell (4,4); the value 4 is excluded.
Do not labor over the rule, just solve the puzzle quickly.



## Hidden Twin Exclusion Rule

**(Mental Exercise):**

This puzzle comes from a 2 x 2 Sudoku Puzzle, illustrating the hidden twin exclusion rule.

The hidden twins are the 2's and 3's in column 1 in Box 1.

Do not labor over the rule, just solve the puzzle quickly.

## Naked Twin Exclusion Rule

**(Mental Exercise):**

This comes from a 2 x 2 Sudoku Puzzle, illustrating the naked twin exclusion rule.

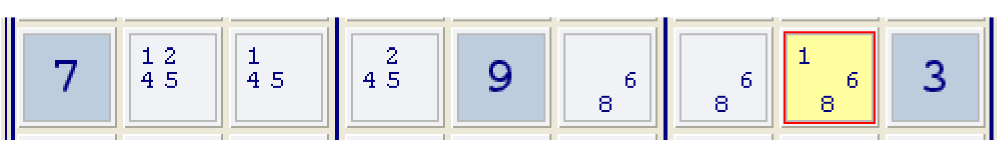
The naked twins are the 2's and 3's in row 3 in Box 3.

Do not labor over the rule, just solve the puzzle quickly.

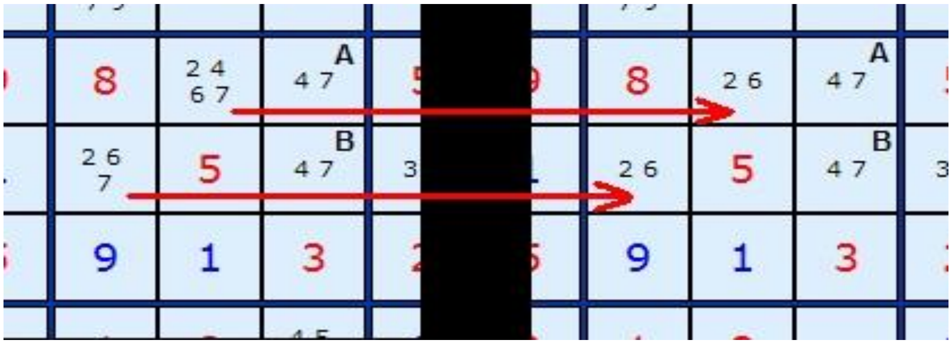
**Naked Pairs:** **A Naked Pair** (also known as **Twins, a Conjugate Pair, a Claim, or a partnership**) is a set of two candidates in two cells that belong to the same Group; i.e. they reside in the same row, box or column. If two cells in a group contain an identical pair of candidates and only those two candidates, then no other cells in that Group could be those values.

**NB: Note Well: this is also known as a Circled Pair under the Parsimonious Force methodology that is illustrated between the two red tabs in this booklet; pages 13-20.**

In the example below, the candidates 6 & 8 in columns six and seven form a **Naked Pair** within the row. Therefore, since one of these cells must be the 6 and the other must be the 8, candidates 6 & 8 can be excluded from all other cells in the row (in this example, it is just the highlighted yellow cell). This identification solves cell (1,8) = 1.

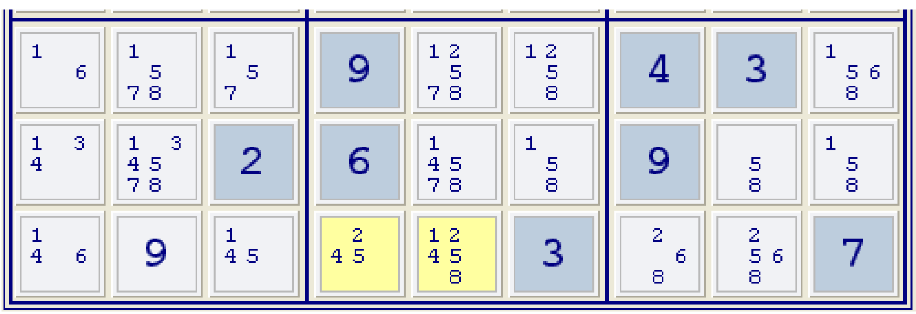


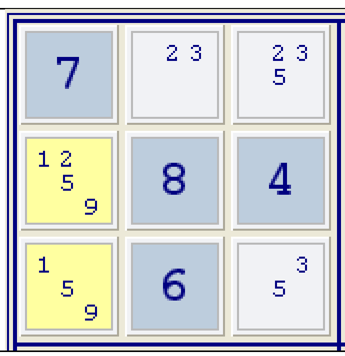
In the following example, there are two 4/7s at **A** and **B**. These pairs preclude all other 4s and 7s in their columns and also in their Boxes. Of course, the **hidden pairs** of 2s & 6s are also **naked pairs**, precluding the all other values in their respective cells. These candidates are removed as shown in the right hand picture.



**Also referred to as Locked Candidates-Rows:** Sometimes a candidate within a Box is restricted to one ro or column. Since one of these cells must contain that specific candidate, the candidate can safely be excluded from the remaining cells in that row or column outside of the Box.

In the example below, the rightmost Box has the candidate 2 only in its bottom row. Since, one of those cells must be a 2, no cells in that Row Group can be a 2. Therefore 2 can be excluded as a candidate from the highlighted cells. {A circled pair in Parsimony Force parlance.}

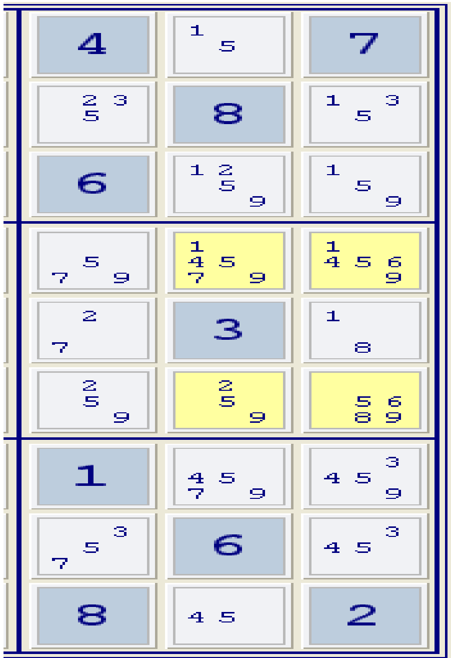


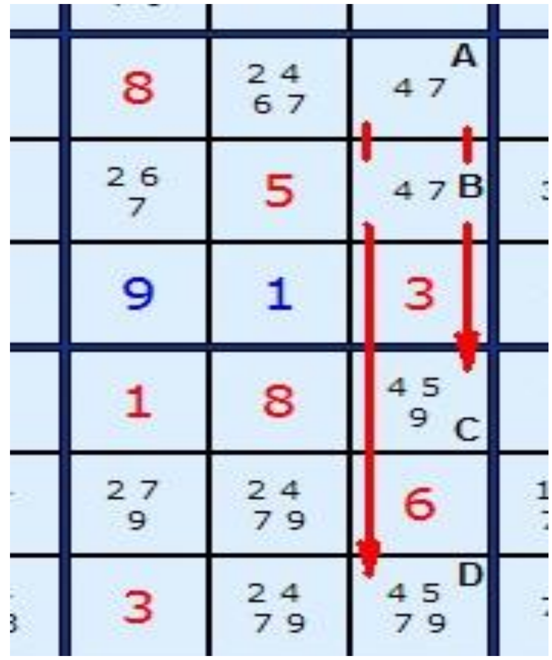


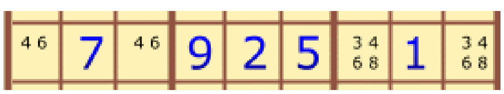
**Hidden Pairs:** If two cells in a group contain a pair of candidates (hidden amongst other candidates) that are not found in any other cells in that Group, then other candidates in those two cells can be excluded safely. In the example on the left, the candidates 1 & 9 are only located in two highlighted cells of a Box, and therefore form a 'hidden' pair. All candidates except 1 & 9 can safely be excluded from these two cells since one cell must be the 1 while the other must be the 9.

**Locked Candidates-columns: In the example directly below, the left column has candidate 9's only in the middle Box. So one of this pair must be in the solution. Therefore, the 9's can be excluded from the yellow cells.**

**The cells at A and B only contain 4 and 7. Therefore, the 4s and 7s at C and D can be removed. Also the other 4s and 7s in the Box can be removed.**



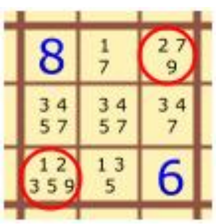




**Naked Pairs: T**wo identical candidates in a particular Group (row, column, or Box)

**I**n this naked pair example, it is safe to eliminate the four and six from the two quads of 3,4,6, and 8. Doing so, leaves two 3,8 pairs. The 3,4,6, and 8 quads are really "**hidden pairs**".

.



**Hidden Pairs**: **I**n this example there is a hidden pair 2 & 9, circled in red. Hidden Pairs (hard to spot) are a pair of candidates that occur in only two cells of a Group (row, column, or box). It is safe to remove all other candidates from the two circled cells so that only the 2 & 9 remain.

**Naked Triples: A Naked Triple** is slightly more complicated because it does not always require **three numbers in each of the three cells**. Any set of three cells in the same Group that contain **any,** but **only** the **same three candidates** is a **Naked Triple**.

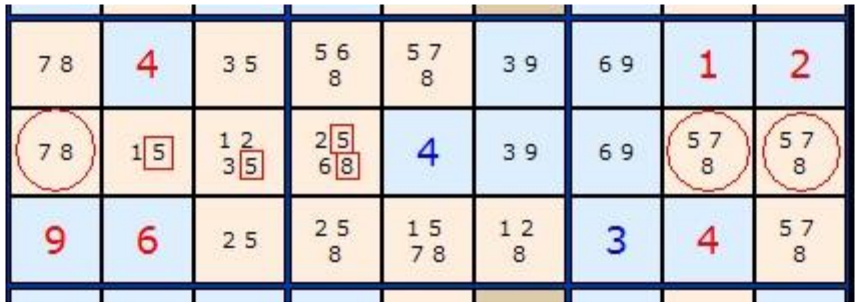
**E**ach of the three cells can have **two or three numbers**, as long they are from the set of the same three numbers. When this happens, the three candidates can be removed from all other cells in their common Group.

**T**he candidates for a Naked Triple for the set 1,2,3 will be one of the following 14 combinations. (123) (123) (123) or (123) (123) (12 or 23 or 13) or (123) (12 or 13 or 23) (23) or (123) (12 or 13 or 23) (13) or (123) (12 or 13 or 23) (12) or (12) (23) (13).

**T**he last case in red is the most interesting since it contains the fewest candidates; and the advanced strategy **XY-Wings uses this formation.**

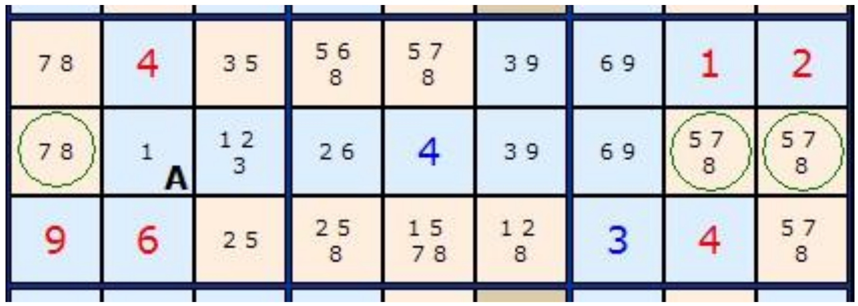
**VERY IMPORTANT CONCEPT:**

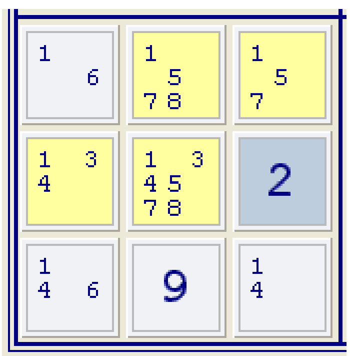
**T**o see a **Naked Triple** in action look at the middle row to the right. We have a Triple in columns 1,8 and 9.



**T**he 5s in columns 2 & 3 & 4, and the 8 in column 4 can be eliminated, while solving column 2 with a 1.

(Note: a continuation of the example above.)



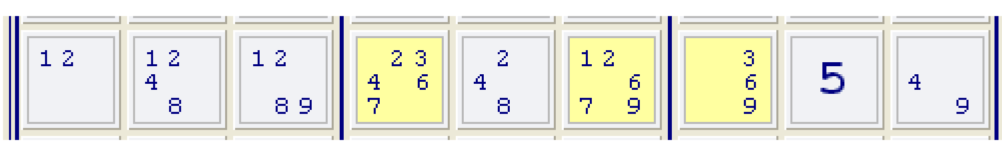


**Naked Triples:** **T**he same principle that applies to **Naked Pairs applies to Naked Triples & Naked Quads.**

**A** Naked Triple occurs when three cells in a Group contain no other candidates other that the same three candidates. **Each of t*he cells making up a Naked Triple does not have to contain all three candidates of the triple*.** If these candidates are found in other cells in the group they can be excluded.

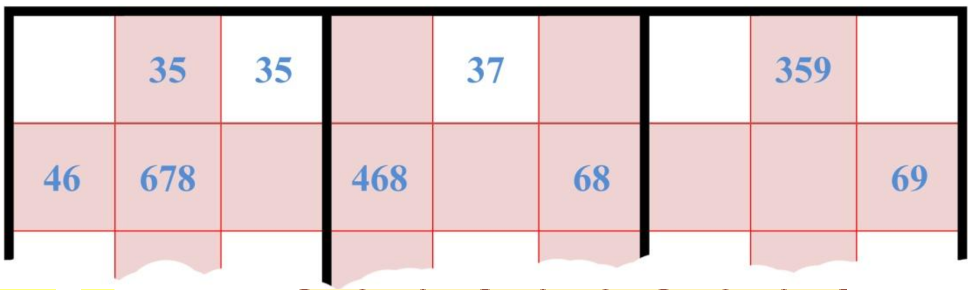
**I**n the example on the left, a Naked Triple is formed by the three gray cells, with candidates 1,6 and 1,4,6 and 1,4. Therefore the candidates 1 & 4 in the highlighted cells can be excluded safely.

**Hidden Triples:** If three candidates are restricted to three cells in a given group, then all other candidates in those three cells can be excluded. In the example below, the candidates 3, 6 and 7 are found only in the yellow cells. Therefore, all other candidates can be excluded from those three cells. ***Hidden triples are generally extremely hard to spot but fortunately they're rarely required to solve a puzzle.***

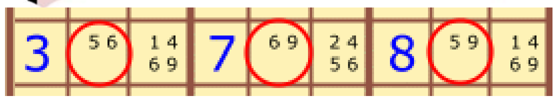


**Twins & Triplets:** The **Naked Pair** of 3s and 5s in Box 1 preclude candidate 3s and 5s in Boxes 2 and 3.

**T**he **Naked Triple** of 4,6,8 in Cells (2,1), Cell (2,4) and Cell (2,6) uses up Cells (2,1), (2,4) and Cell (2,6) so the candidates 4,6, and 8 cannot appear elsewhere in row 2.

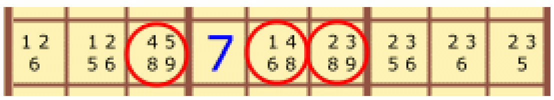



**Naked Triples: Here is an example of a Naked Triple with the candidates 5,6,9.**




Study this illustration carefully. Note that the circles only contain combination pairs of 5,6,9. But there are **14 combinations of 5,6,9** taken two or three at a time, and any of these combinations can be included in the circles. See combinations in middle of page 27.

Also note that when the 5,6,9s are safely removed from the third, sixth and ninth cell, we find a naked pair of 1,4s in the third and ninth cell, and we also find the naked singleton, 2, in the sixth cell.

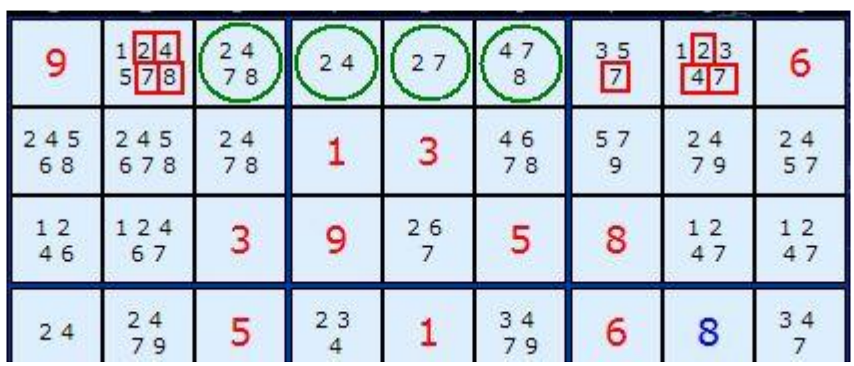


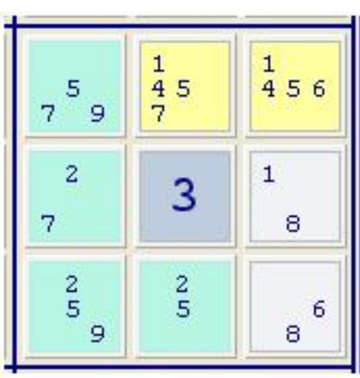

**Hidden Triples** are similar to **Naked Triples**, but are more difficult to spot.

The Hidden Triples in the red circles are 4,8,9 and they occur as 4,8,9 and 4,8 and 8,9 respectively. The candidates 1,2,3,5 can be removed from the circles.

**Naked Quads** are rare, especially in its full form but still useful if they can be spotted. The same logic from Naked Triples applies. Quads should not be forgotten; but they are of limited usefulness, (until you need it!)

The four green circles mark a Naked Quad in the top row. Therefore, the numbers 2,4,7,8 marked with the red cells can be safely eliminated.




**Naked Quads:** A Naked Quad occurs when four cells in a Group contain no other candidates other that the same four candidates. Each of ***the cells making up a Naked Quad does not have to contain all four candidates of the triple.*** If these candidates are found in other cells in the Group they can be excluded.

In the example on the left, the candidates 2, 5, 7 & 9 in the Aqua cells form a **Naked Quad**. Therefore candidates 5 & 7 that are in the highlighted cells can be excluded.

**Hidden Quads:** If four candidates are restricted to four cells in a given group, then all other candidates in those four cells can be excluded.

***Hidden Quads are very rare, which is fortunate since they're almost impossible to spot even when you know they're there.***

Try and spot the Hidden Quad in the row below. (Hint: 2,6,7,9)(cells 1,2,8,9)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 6 9 | 2 7 9 | 3 8 | 1 3 4 5 8 | 3 4 5 | 1 3 4 5 | 4 5 8 | 2 5 7 | 4 6 7 8 |

**Three in a Row in a Box:** Cell (2,5) must contain a 5 because the 6,7,8 in Box 3 force a naked pair of 5's in the top row of Box 3; i.e. cell (1,7) and (1,9).

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | 1 | |
| | | | 2 | | 4 | 6 | 7 | 8 |
| | 5 | | | | | | | |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | 9 | | |
| | | | | 9 | | | | |
| 9 | | | | | | | | 7 |
| | | | 6 | | 7 | | 2 | |
| | | | | | | 5 | | |

**Three of a Kind in Boxes:** Cell (8,9) must contain a 9 because the 9 in Cell (6,5) forces a naked pair of 9s in the row 9 of Box 8. **This requires a bit of thought.**

**Filling in the Gaps:** Find a starting point like Cell (2,9)=3; Cell (2,6)=8; Cell (2,5)=4; etc, etc.

When analyzing a Group, (Box, column, row), guess at what candidate numbers are outstanding, rather than count 1,2,3………,9. One must develop an "intuitive" awareness of the missing number. It takes a wee bit of practice.

When looking at the three numbers 1,2,3 we immediately know that the 2 is missing in 1,_,3 without thinking. **The same skill can be developed when looking at 4,5,8,_,7,2,_,9,3. Just continue to practice, or shuffle and deal out 7 cards.**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 8 | 5 | 3 | 1 | 6 | 9 | 2 | 4 | 7 |
| 1 | 6 | 7 | 2 | | | 9 | 5 | |
| 9 | 2 | 4 | 5 | | | | 8 | |
| 7 | 9 | 5 | | 3 | | 1 | | 8 |
| 6 | 3 | 2 | 8 | 5 | 1 | 4 | 7 | 9 |
| 4 | 1 | 8 | | 9 | | | 2 | 5 |
| 3 | 4 | 9 | | | 5 | 7 | | |
| 5 | 7 | 1 | | | 4 | 8 | 9 | 6 |
| 2 | 8 | 6 | 9 | 1 | 7 | 5 | 3 | 4 |

**Naked Quads:** In the following example, the **Naked Quads** are 3,5,6,8. So the candidate 3 can be removed from column 4 leaving a Naked Pair of 4,9.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 56 8 | 1 | 49 | 34 9 | 2 | 68 | 7 | 35 | 35 6 |

**Hidden Quads:** In the example at the right, the **Hidden Quads** are 1,5,6, and 8. It is safe to remove the extra digits (3,4,7,9) from these four cells.

When there are a lot of candidates showing from the surrounding rows, columns, and Boxes, **Hidden Quads** are almost impossible to spot.

Why does this Work?

| |
|---|
| 58 9 |
| 27 |
| 16 8 |
| 23 49 |
| 23 4 |
| 23 479 |
| 14 6 |
| 34 7 |
| 35 67 |

## Uniqueness Elimination:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 89 | 5 | 89 | 3 | 6 | 7 | 4 | 1 | 2 |
| 4 | | 3 | 9 | | | 5 | 6 | 8 |
| | 6 | 1 | 4 | | 8 | 9 | 3 | 7 |
| | 9 | | 7 | | | | | |
| | | | 6 | | 9 | | | |
| | | | | | 2 | | 7 | 9 |
| 6 | 1 | 7 | 5 | 9 | 3 | 8 | 2 | 4 |
| 389 | 4 | 89 | 2 | | 6 | 7 | 5 | |
| 5 | 38 | 2 | | 7 | 4 | | 9 | |

Evokes the Golden Rule: Each Sudoku puzzle must have one and only one solution.

This is subtle. Cell (2,9) must be an 8. If it were the 3, then we would have an illegal X-wing in the other four shaded pink cells. Cell (1,1) could be an 8 or 9 and it doesn't affect any of the other non-shaded cells. This means the puzzle violates the Golden Rule.

This is an example of elegant reasoning and is an important tool in many instances. This was found in Sudoku Master Class by Thomas Sheldon.

| | | |
|---|---|---|
| 6 | 47 | 19 |
| 3 | 28 | 12 8 |
| 5 | 47 | 29 |
| 7 | 28 | 28 |
| 9 | 1 | 6 |
| 4 | 3 | 5 |

**What candidate must reside in Cell (5,3); i.e. the circled cell containing the candidates 1,2,8.**

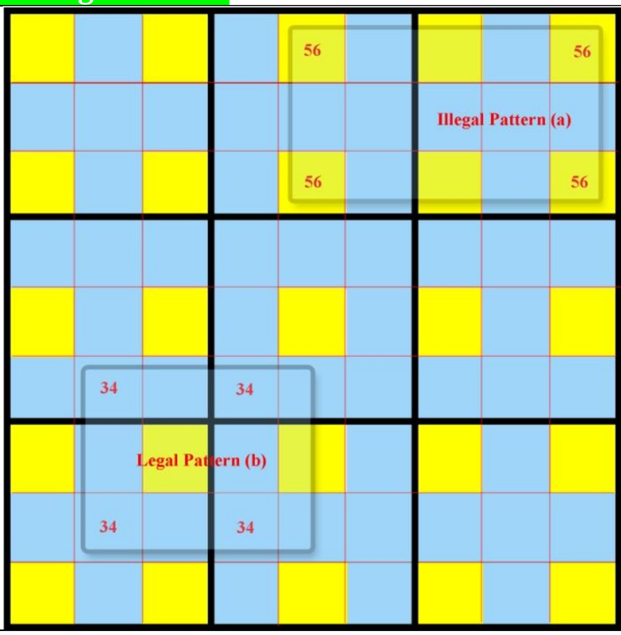
**Evokes the Golden Rule: Each Sudoku puzzle must have one and only solution. Otherwise the four cells would be indeterminable**

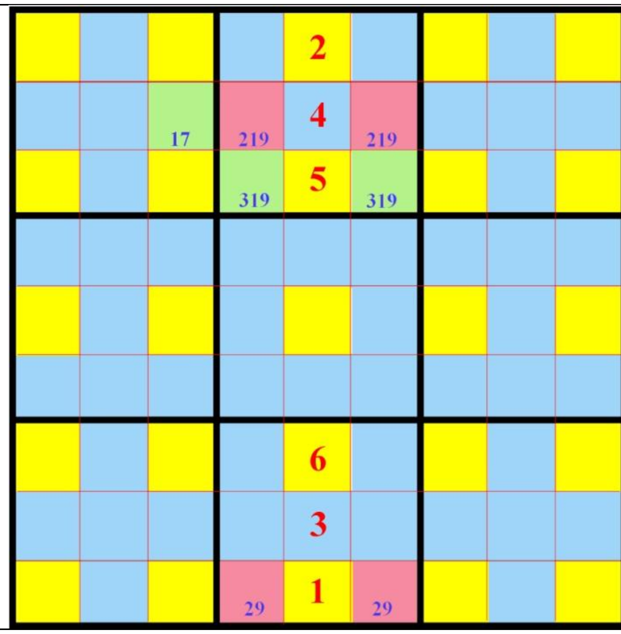
## Subtle Uniqueness Patterns

The left example below shows both an illegal X-Wing and a Legal X-Wing. **Can you see the difference?** The naked pairs of 5,6 in the illegal X-Wing are confined to two Boxes. Swapping the 5 and 6 does not change the content (arrangement of the other numbers 1,2,3,4,7,8,9) of Boxes 2 and 3. Thus we have **two solutions to the same puzzle.**

In the second example, the naked pairs of 3,4 are spread over four Boxes. So swapping the 3 and the 4 changes the contents (or arrangement of the numbers 1,2,5,6,7,8,9) in each of the four Boxes 4, 5, 7 and 9, thereby providing a unique solution.

The pattern in the right hand puzzle is sometimes referred to as **Roof & Floor**. When the author originally saw this pattern, the Roof was where the Floor was, and vice versa. So it was decided to forget this analogy. However, if any of the **green cells** contain a candidate 1, then the **pink cells** form an illegal X-Wing, like the example on the left. So the 1's can be safely **eliminated from the three green cells.**




## Gordonian Logic:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 6 | 5 | 1 | 4 | 8 | 2 | 9 | 7 | 3 |
| 27 | 24 | 47 | 379 | 65 | 3579 | 65 | 1 | 8 |
| 8 | 3 | 9 | 71 | 615 | 75 | 65 | 4 | 2 |
| 4 | 9 | 8 | 2 | 7 | 6 | 1 | 3 | 5 |
| 5 | 7 | 3 | 91 | 41 | 49 | 2 | 8 | 6 |
| 1 | 6 | 2 | 5 | 3 | 8 | 4 | 9 | 7 |
| 9 | 24 | 47 | 37 | 245 | 3475 | 8 | 6 | 1 |
| 27 | 1 | 6 | 8 | 24 | 47 | 3 | 5 | 9 |
| 3 | 8 | 5 | 6 | 9 | 1 | 7 | 2 | 4 |

This technique was found in ***Mensa Guide to Solving Sudoku*** and discovered by **Peter Gordon**.

The blue cells contain a set of **Naked Pairs** so the 5 and 7s can be eliminated. The 4s are Claiming the lower blue cells so the 4s can be eliminated in the remainder of the Box.

The pink cell must be a 1. If not a 1, then we have an Illegal X-Wing of 6s and 5s in Boxes 2 and 3. Since either pair of cells can be 6s and 5s, there is no possible solution, violating the principle that there is but one solution.

This is called **Gordonian Logic** and is very similar to the **Uniqueness Principle** in the preceding example.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 67 | 3 | 4 | 87 | 687 | 5 | 92 | 92 | 1 |
| 9 | 5 | 8 | 1 | 3 | 2 | 4 | 7 | 6 |
| 67 | 2 | 1 | 4 | 9 | 67 | 3 | 8 | 5 |
| 8 | 6 | 3 | 5 | 17 | 917 | 917 | 4 | 2 |
| 1 | 7 | 9 | 6 | 2 | 4 | 8 | 5 | 3 |
| 5 | 4 | 2 | 987 | 817 | 3 | 917 | 6 | 97 |
| 2 | 8 | 5 | 3 | 4 | 97 | 6 | 1 | 97 |
| 3 | 1 | 6 | 29 | 5 | 8 | 927 | 92 | 4 |
| 4 | 9 | 7 | 2 | 61 | 61 | 5 | 3 | 8 |

**Many of these Uniqueness pairs or Gordonian pairs are easy to spot.**

**Cleary in this example if Cell (8,7) is not a 7, then the four colored cells would produce a multiple solution.**

**In the first case cell, (1,7) would contain a 9, and in the second case cell (1,7) would contain a 2, and this would violate the golden rule, that only one solution per puzzle.**

## Rule of Uniqueness – Rectangle & XY-Wing

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| | 78 | | | | | 89 | | |
| | | | | | | | | |
| 567 | 56 | | | | | 78 | | |
| | 7X | | | | | | | |
| | 7X | | | | | | | |
| 7X | | | | | | | | |
| 56 | 567 | | | | | | | |
| 7X | | | | | | | | |

**We will learn more about XY Wings later.**

**One must have an eagle eye to spot the four pairs of 56's in the yellow cells. This is a dead give away for a potential violation of the Uniqueness Rule – "Every puzzle must have but one solution."**

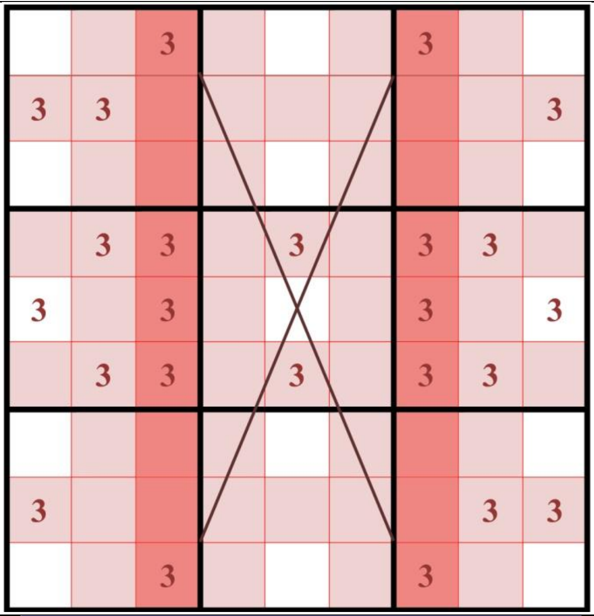
**For certain then, either Cell (7,1) or Cell (8,2) must contain a 7. Therefore the 7s in the pink cells can be eliminated. The Xs in the pink cell can be any of the numbers from 1 thru 4.**

**Finally Cell (2,7) must be 9. In either case, whether Cell (7,1) or Cell (8,2) contains a 7, the subsequent analysis will force Cell (2,7) to contain a 9.**

## The Franciscan Elimination

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 56 | | | | 67 | | 57 | | |
| 56 | | | | 67 | | 578 | | |
| | | | | | | | | |

**The Franciscan Elimination, credited to Francis Heaney, is a simple, clear example of the Uniqueness Principle. The pink cell must be an 8, otherwise the puzzle is indeterminate. The top row showing could be 5,6,7 or 6,7,5; i.e. two solutions.**



**The X-Wing**: This is a FILTERED graphic showing all possible 3s in a hypothetical puzzle.

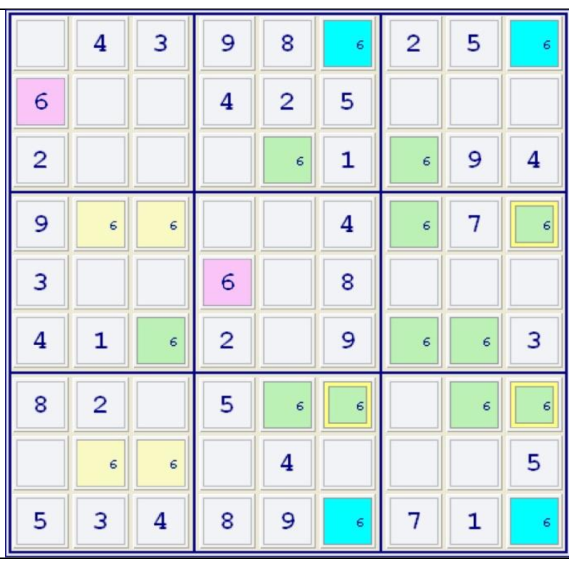
Cells (1,3) and (9,7) must contain a 3 or Cells (1,7) and (9,3) must contain a 3. Thus the 3s in Box 4, column 3 and Box 6, column 7 can be safely eliminated.

The name is from the pattern shown below. It has been stated in the book ***Extreme Sudoku for Dummies*** that the inventor was a fan of Star Wars X-Wing Fighters.

This illustration provides a simple, clear example of an X-Wing. An X-wing requires a single candidate to appear in four cells that form a rectangle as shown with the four green cells in the illustration which contain the candidate 4.

Connect the two diagonals of the rectangle. One of the two pair of diagonals must be the correct solution. The 4's in the purple cells can be discarded.

**NB**: obviously, the **green cells** contain other candidates beside the 4s so it requires an eagle eye.



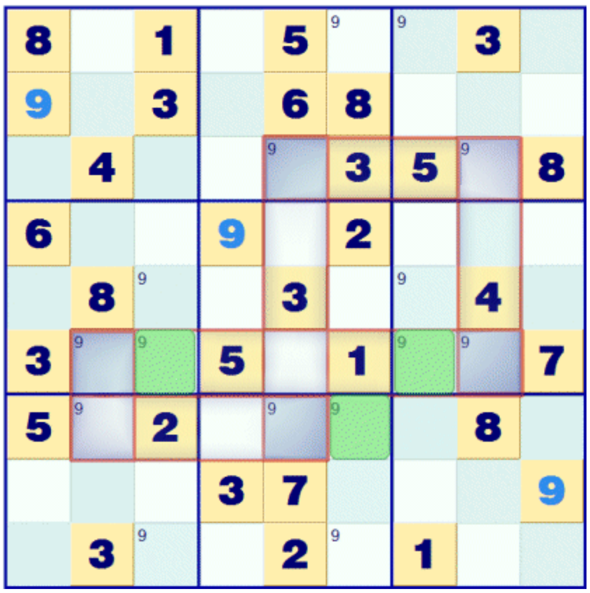
**X-Wing:** this example has been over simplified by FILTERING out the other candidates in the blue, green and yellow cells.

The **blue** cells form an X-Wing. Thus, it is safe to eliminate three 6s in columns 2 and 9.

This example is quite interesting because the yellow cells form an illegal X-Wing. The reason: Cell (1,9) must be a 6 by virtue of the Uniqueness Rule; otherwise there is more than one solution to the puzzle because the yellow illegal X-Wing would be indeterminate

**A Swordfish** requires a single candidate (in the following illustration a 9) to appear in only two cells in either 3 rows or 3 columns. This example below is a Column Swordfish. That means there are only two possible 9 candidates in columns 2,5, and 8. Additionally there must be at least 6 rows and columns that interlock in several rectangles as shown in the illustration on the below.

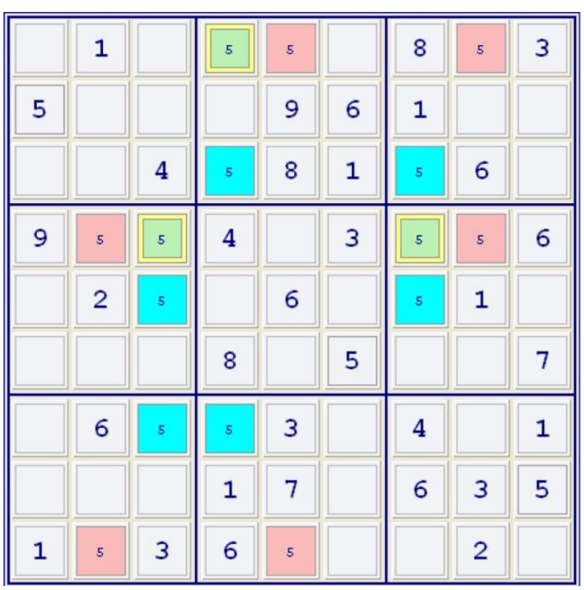
This illustration below provides a simple, clear example of a Swordfish – the blue-gray cells which form two interlocking rectangles



One of the 9s in each of the highlighted rows or columns must be correct. Thus, any other 9s in the columns can be eliminated. **See green cells**.

**NB**: this example was FILTERED; the **green and blue grey cells** contain other candidates beside the 9s but are not shown.

The illustration also serves to illustrate a **Chain**. For example if cell (3,5) is true, then (3,8) is false, and (6,8) is true, and (6,2) is false, and (7,2) is true, and (7,5) is false and finally cell (3,5) is true and consistent.



**Swordfish:** The Swordfish pattern is a variation on the "X-Wing" pattern above.

In this example above, filtering has been applied so only candidate 5’s is visible.

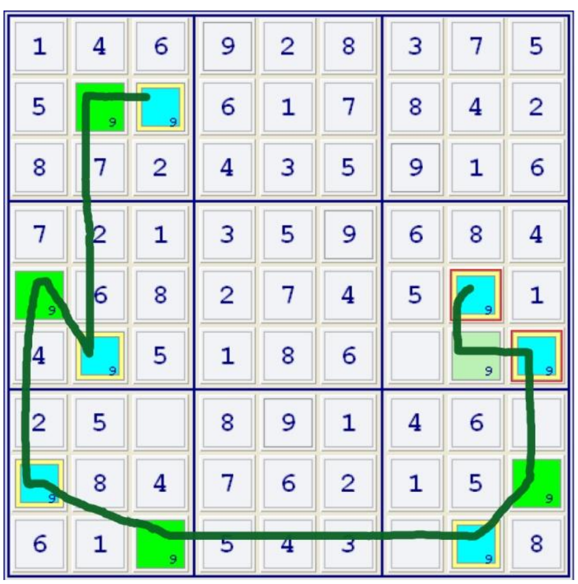
These blue cells are an example of a row Swordfish, since rows 3, 5 and 8 only contain 2 cells with candidate 5. The pink cells are a reciprocal column swordfish, by coincidence.

In either Swordfish, the green cells with yellow borders columns 3, 4 and 7 can be eliminated.

## Colors and Chains: Solving with Colors and Chains:

Of special interest are candidates that reside in only two cells in a given group (a row, column or box). These pairs of cells are said to have a **"conjugate' relationship."** If one cell is **(true)** then the other must be **(false)**. We don't yet know which is true and which is false. But in our minds eye we can visualize this relationship using two colors (we have arbitrarily chosen blue and green in the example, and we are starting with the assumption that **blue is true).**

In the illustrations below, the graph has been filtered to eliminate 2s, 3s, 7s, and 9s that may be possible candidates in the **green** and **blue** cells. In the Chain example, the initial assumption was that the blue Cell (2,3) was True.


Typically there are a number of **'conjugate pairs'** present at any given time. Usually these conjugate pairs can link with other conjugate pairs forming a chain of alternate true/false cell states, and these chains may expose candidates which can be excluded safely.

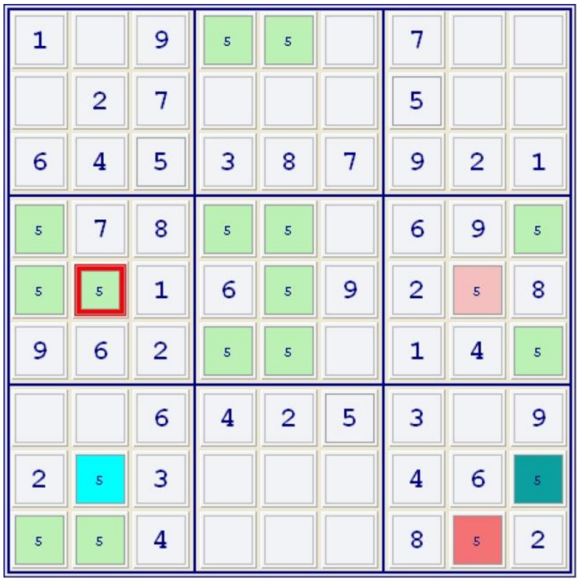
To establish a link, the pairs must be in the same Group (row, column or box.) Study the illustration on the left and it will become clear that the links are always in the same Group.

Whenever two cells in a conjugate chain have the same color and also share the same group, that color must be the 'false' color since each group can only have one of either color.

**In lieu of coloring, cells can be linked using a True-False chain as shown in the illustration. The chain that is shown is one of many possible chains that can be drawn.**

**The cardinal rule is that the cells linked must be from the same Group . All of the possible chains result in a consistent conclusion – an affirmation or contradiction. When the chain ends with two cells that are both true or both false in the same Group, then the initial assumption is incorrect.**

**In Box (2,3) both Cells (6,9) and (5,8) cannot both be True (blue).**



This illustration of **Coloring or Chaining Conjugate Pairs** is straightforward.

Column 8 contains only 2 candidate 5s that are therefore conjugate pairs in shades of red.

Row 8 in a similar fashion contains only 2 candidate 5s that must be conjugate pairs in shades of blue.

Therefore the **red** and **blue** cells in Box 9 must also be conjugate pairs forming a chain of four cells. Cell (5,2) shares the **intersection** of the pink and light blue cells and can be eliminated.

## Colors, Chains, Swordfish

A **rose** by any other name is still a **rose**.

The illustration shows a pattern that can be solved by Colors, by True-False, by Chain or by a Swordfish. All lead to the same Elimination of the Red Cell (7,9)

**MORE ON CHAINS:** The next few pages are tedious to read and follow. Patience and re-reads are required. Once understood, **Chains** become a powerful tool for difficult puzzles. There are two primary advantages to identifying and using chains.

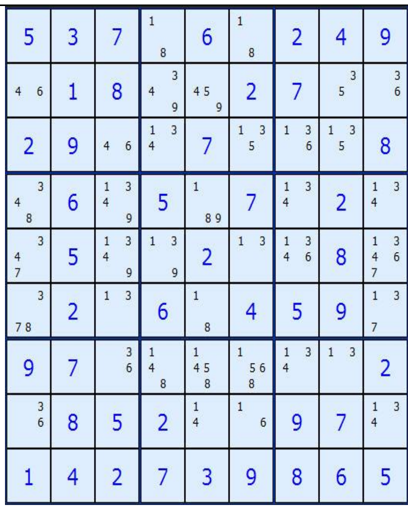
**First**, common candidates that share the same **Group or Sudoku Unit**, which contain both the Starting and Ending cells, can be eliminated. The cells we are discussing are illustrated in the third grid on the far right. The starting cell is **green** and the ending cell is **yellow**. The **green** and yellow cells both share cell (2,5) and cell (3,8). In the following example, the 5's in cells (2,5) and (3,8) can be eliminated because they are in the **Sudoku Units** of both the **Starting and Ending Cells**.

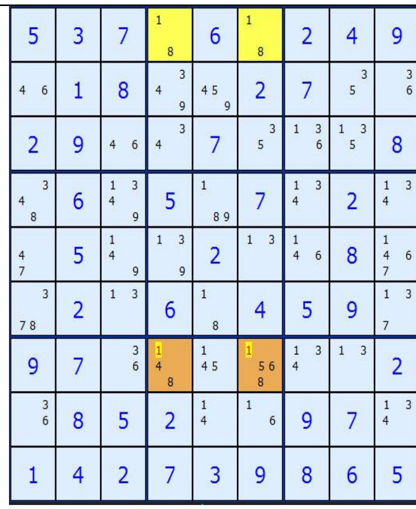
**Caution**: at first blush, it would appear that all the 5's and 3's in Boxes 2 and 3 can be removed since the **green** and **yellow** cells look like naked pairs. They are not naked pairs since they do not share a **common Group**. **{No, this does not contradict previous paragraph.]**

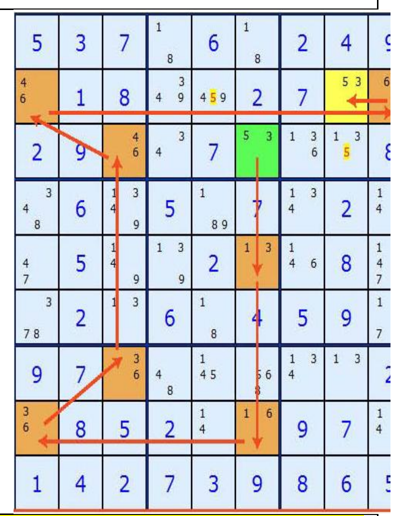
**Here** is the Logic: If the **green cell** is 5, then 5 can be eliminated from Cells (2,5) and (3,8). If the green cell is not 5, then it must be a 3. In this case, **the brown cells**, in turn would have to be 1, 6, 3, 6, 4, 6, 3 and finally 5 in the **yellow cell**. The logic does not work for a starting value of 3 in the **green** cell because the **ending yellow** cell does not end up with a 3.

**The Second** advantage for **Chains**: once any one of the candidates for any of the **green, yellow or brown cells** is determined, the others follow like dominoes. So the **Chain** actually identifies two series, one of which is the **TRUE** series. If we assume cell (3,6) is **TRUE** for 3, then the series is 3,1,6,3,6,4,6,3,5. But, alas, it starts with 3 and ends with 5 which not consistent. If cell (3,6) starts with 5 as **TRUE**, then the series is 5,3,1,6,3,6,4,6,5 which is the **TRUE** series.

Some eagle eyes might contend that cell (2,8) should be a 3, and not a five sinc a 3 follows logically from cell (2,9). The answer is simple, the 3 is not force from its Sudoku Unit, and so the chain maker has a choice of 3 or 5.



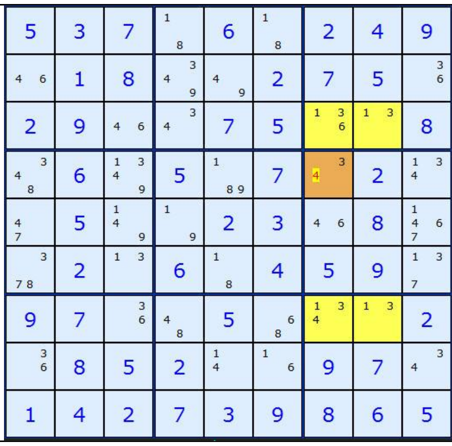


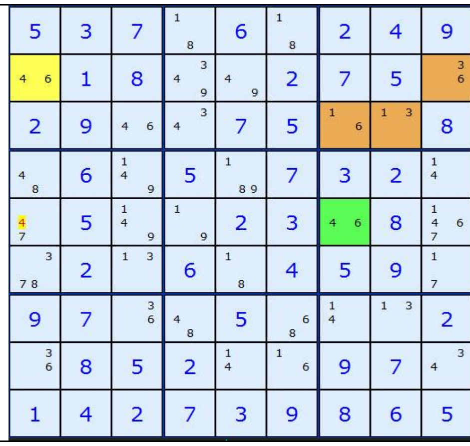


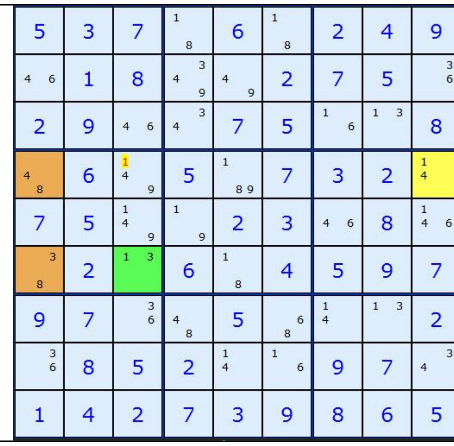
**Note: the three grids above and the five grids below are the same puzzle showing steps in its solution, starting in the upper left above and ending in the lower right below.**

**The steps are outlined below. This is tedious and will test both your patience and accuracy, both of which are required to solve difficult puzzles.**

**[1]** The **naked pair** of 1s in (3,7) & (3,8) eliminates the 1s in (3,4) & (3,6). **[2]** The **pointing pair** of 3s in (5,4) & (5,6) eliminates the remaining four 3s in column 5. In illustration 2, the pair of 1s are eliminated in (7,4) & (7,6) because of **Uniqueness.** **[3]** The **XY-chain** in right graph above**,** starting in (3,6), ending in (2,8) eliminates the 5s in (2,5) & (3,8). **[4]** And leaves **singletons** of 5s in (2,8), (3,6) & (7,5) and a 3 in (5,6). [5] The **pointing pairs** of 1s in (8,5) & (8,6) eliminates the 1 in (8,9). [6] The **triplets** of 1 in Column 9 eliminate the 1s in (4,7) & (5,7). [7] In the left graph below, (4,7) must be a 3 because of **Uniqueness**. [8] the **XY-chain** in the middle graph below, starting in green and ending with yellow, eliminates the common 4 in (5,1) forcing a 7 in (1,5) & (6,9). [9] The **XY-chain** in the right graph below, starting in green and ending with yellow, eliminates the common 1 in (4,3). [10] The **XY-chain** in the lower left graph, starting with green and ending with yellow, eliminates the common 9 in (4,5). [11] this results in **singletons** (2,5)=9; (4,3)=9; (8,5)=4; (8,6)=1, etcetera to a complete solution







**T**he **moral** behind this exercise is that if one is patient and accurate in using the **Detail Markup**, puzzles can be solved in the most interesting of ways.

**T**he author does not use the **Detail Markup** method but enjoys solving difficult puzzles on the computer, which generates the Detail Markup if and when his puzzle stalls.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5 | 3 | 7 | 1 | 6 | 8 | 2 | 4 | 9 |
| 4 | 1 | 8 | 3 | 9 | 2 | 7 | 5 | 6 |
| 2 | 9 | 6 | 4 | 7 | 5 | 1 | 3 | 8 |
| 8 | 6 | 9 | 5 | 1 | 7 | 3 | 2 | 4 |
| 7 | 5 | 4 | 9 | 2 | 3 | 6 | 8 | 1 |
| 3 | 2 | 1 | 6 | 8 | 4 | 5 | 9 | 7 |
| 9 | 7 | 3 | 8 | 5 | 6 | 4 | 1 | 2 |
| 6 | 8 | 5 | 2 | 4 | 1 | 9 | 7 | 3 |
| 1 | 4 | 2 | 7 | 3 | 9 | 8 | 6 | 5 |

**A**n excellent internet site for computer analysis of difficult puzzles of all shapes and sizes is found at a site maintained by **Andrew Stuart**. The author believes it is one of the best sites on the Internet.

SUDOKUWIKI.ORG
Strategies for Number Puzzles of all kinds

http://www.scanraid.com/sudoku.htm

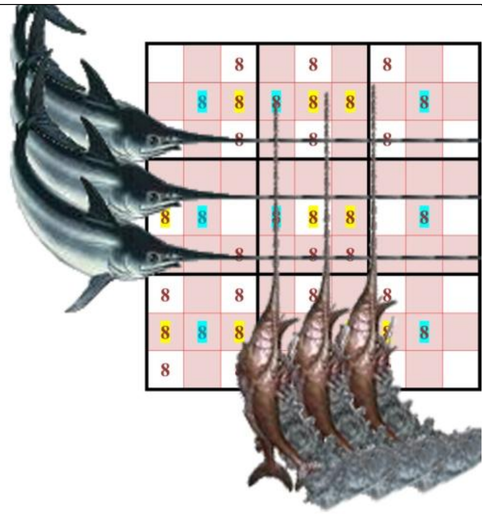
## XY-Wing: Not to be confused with X-Wing. Requires 3 candidates: X, Y, & Z

**A**ll 3 cells have exactly 2 candidates.

**A**ll 3 cells share the same 3 candidates in the form – XY, XZ, YZ.

**O**ne cell (the Y 'stem' with candidates XY) shares a group with the other 2 cells (Y 'branches' with candidates XZ & YZ).


**T**he **XY-Wing** is not rocket science. **Just think of a Bee**. The **"Y" stem** is really the body of a Bee with a left wing (XZ) and a right wing (YZ). The body, or the "Y" stem contains XY and is really the cell in the middle of the wings. In the example below, the body is the Pink Cell containing candidates 8 and 2 and the Wings are the Blue Cells with 5 being the Z candidate..

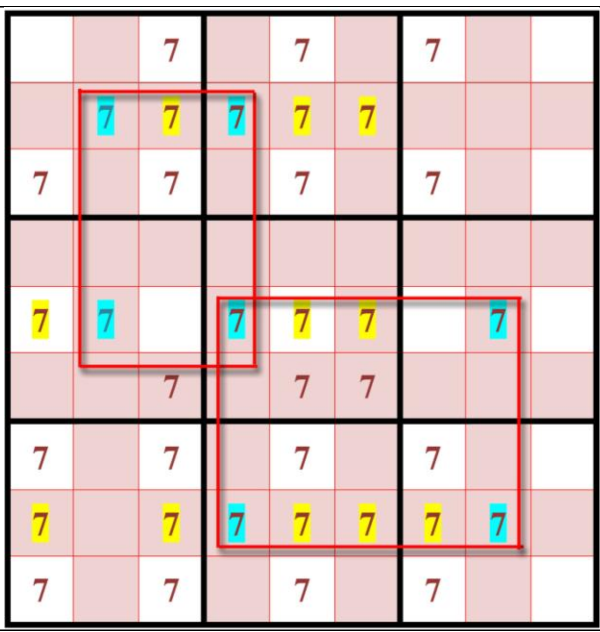


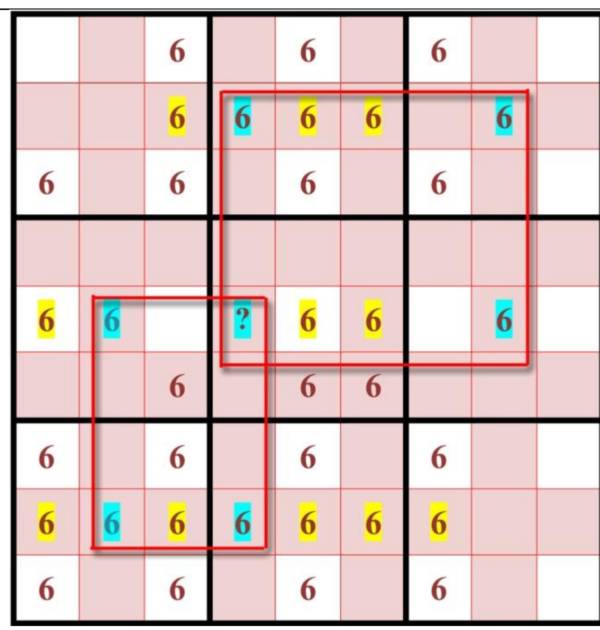
How Swordfish got its name

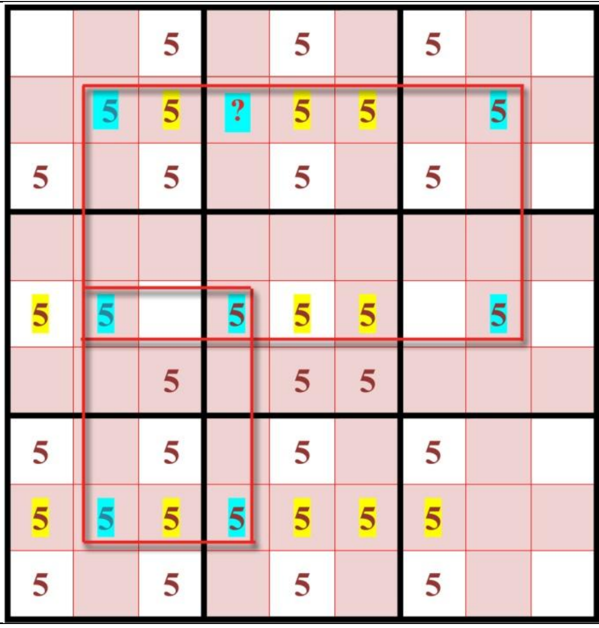
**Swordfish Patterns:** An X-Wing involves 4 candidates sharing two rows and two columns. And the solution is governed by an X pattern.

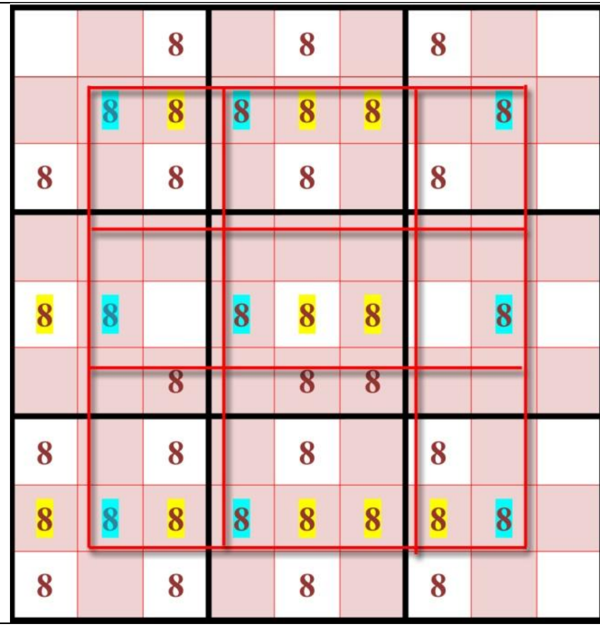
A Swordfish, on the other hand, always involves 3 rows and 3 columns. A Perfect Swordfish requires 9 candidates, symmetrically aligned in the 3 rows and 3 columns as shown in the lowest right column ; the grid with the 8s. In this example, each intersection contains a Blue 8. Each of these row/columns must contain one of the Blue 8's for a total of /three 8s. So we can eliminate for certain, the 11 yellow 8s shown in the example.

In all other cases, the Swordfish pattern must contain 8, 7 or 6 candidates involving 2 intersecting rectangles. Three of the examples are shown below, and other rectangle combination can be drawn, usually with a pattern of 3,3,2 or 3,2,3 or 2,3,3 or 3,2,2 or 2,3,2 or 2,2,3 or 2,2,2.





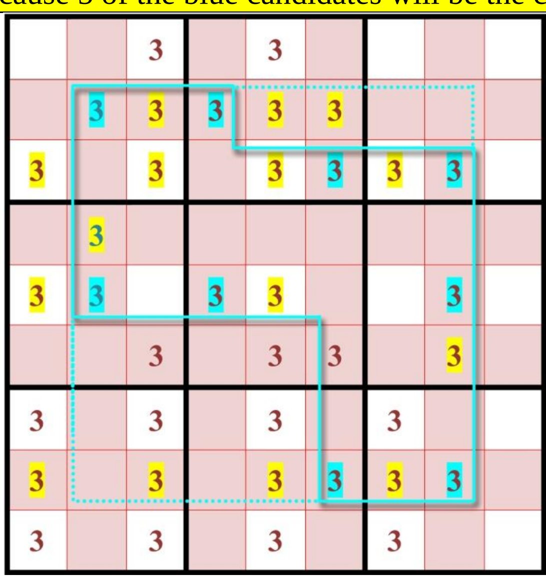


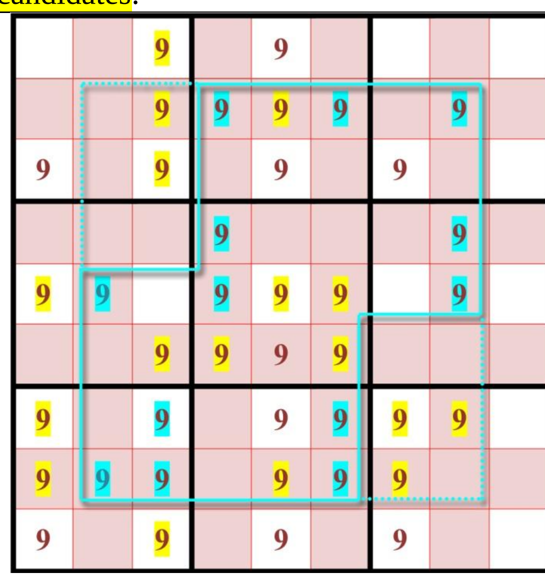


## Jellyfish and Squirmbag are extensions of the Swordfish.

**Jellyfish** contain 4 rows or 4 columns. Its shape actually can by 4x4, or 3x4, or 4x3. At least 2 candidates must be in each row or column. It can contain as few as 8 and as many as 16 }candidates. **The illustration shows 9 candidates in blue** { HARD TO READ**}**. All the yellow candidates which are in a row or column of the blue candidates can be safely eliminated because 4 of the blue candidates will be the correct candidates.

**Squirmbag** contain 5 rows or 5 columns. Its shape actually can be 5x5, 4x5, 3x5, 5x4 or 5x3. It can contain as few as 10 and as many as 25 candidates. All the yellow candidates which are in a row or column of the **blue candidates{HARD TO READ}** can be safely eliminated because 5 of the blue candidates will be the correct candidates.





**Eppstein Bi-Value Cycle:** Professor David Epstein is at Cornell U; his papers on repetitive and non-repetitive bilocation cycles that can be applied to Sudoku can be found at his website: http://arxiv.org/abs/cs.DS/0507053

The Bi-Value Cycle takes a lot of words to explain, but is easily understood. It is very similar to chaining, but with a restriction: when a pair of cells are linked with a common candidate, that candidate cannot appear elsewhere in the Group. On the other hand, the linking cells can have more than two candidates which leads to the power of the cycle. If and when the cycle is completed, the additional candidates in cells with more than two candidates, can be eliminated, enabling forces elsewhere in the Grid.

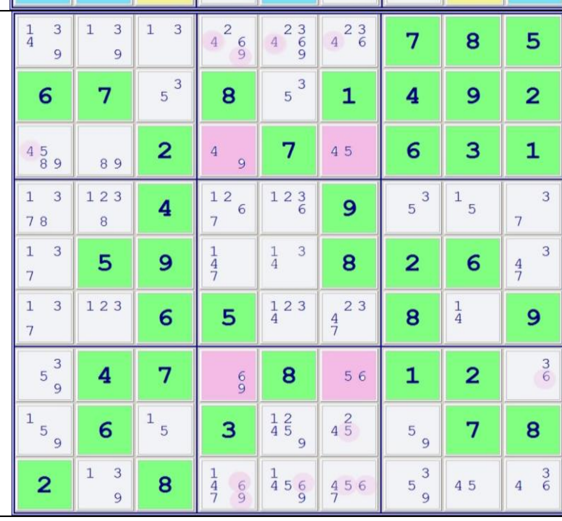
Serious students can find more information at the professor's website, or in the ***Mensa Guide to Solving Sudoku*** by **Peter Gordon and Frank Longo.**

## TRUE FALSE CHAINS:

If the 6 in cell (9,8) is assumed to be True, the 9 in the **Tan** Cell (5,4) must be true, and the chain is consistent. Otherwise the chain is not consistent.

## XY-Chain or Naked Quad:

**P**ink cells represent four cells with four and only four unique candidates. Without further adieu, the quad eliminates many candidates covered with the pink dots that are in the respective **Sudoku Units** of each of the four pink cells.



**End Game Strategies**: **The place for educated guessing.** **H**ere are a few ideas for end game considerations. Guessing is a waste of time, since other alternatives are available. When you are really stuck, and want to guess, immediately check your answer. Alternatively, there are several outstanding web sites with on-line Sudoku Solvers. Or you can download the ***Simple Sudoku Solver*** by **Angus Johnson**. It is attached as an executable program to this PDF document on disk. **W**hen you are in the end game, let's say with 20 or less remaining cells, and a lot of familiarity with the remaining possible candidates, as shown in the following example, educated guessing makes sense. Here are a few illustrations where it does make sense.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 7 | 6 9 | 5 9 | 8 | 5 6 | 4 | 1 |
| 8 | 1 5 | 6 | 3 | 4 | 1 5 | 2 | 9 | 7 |
| 4 | 1 5 | 9 | 1 6 | 2 | 7 | 8 | 5 6 | 3 |
| 3 | 6 | 2 | 5 | 8 | 4 | 7 | 1 | 9 |
| 5 | 7 | 1 | 2 | 6 | 9 | 4 | 3 | 8 |
| 9 | 8 | 4 | 7 | 1 | 3 | 5 6 | 2 | 5 6 |
| 1 | 2 | 3 | 4 | 7 | 5 6 | 9 | 8 | 5 6 |
| 6 | 9 | 5 | 8 | 3 | 2 | 1 | 7 | 4 |
| 7 | 4 | 8 | 1 9 | 5 9 | 1 5 6 | 3 | 5 6 | 2 |

**S**tart anywhere on the 5's and use color, or true or false. Eventually you will find a contradiction, as in cell 9,6

**W**hen linking, remain in the cells in a Sudoku Unit. Try using T or F.

**O**r guess on 5 or 9 in cell (1,5). Both will produce a 6 in cell (9,8) which breaks open the puzzle. The 6 in cell (9,8) is correct, independent of the value in cell (1,5).

**2 works, 4 doesn't; or Start Anywhere**

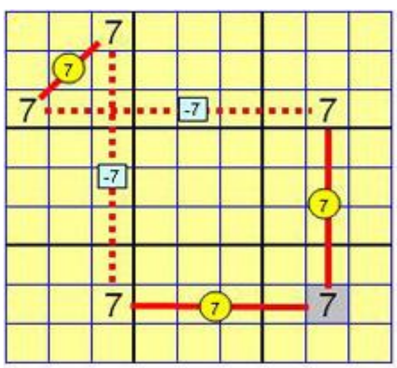
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 4 | 3 6 | 5 | 1 | 7 | 8 | 2 3 | 9 | 4 6 |
| 4 9 | 3 6 | 7 | 6 9 | 3 4 | 2 | 8 | 5 | 1 |
| 2 4 9 | 8 | 1 | 6 9 | 3 4 | 5 | 2 3 | 7 | 4 6 |
| 6 | 9 | 3 | 5 | 2 | 7 | 1 | 4 | 8 |
| 5 | 2 | 8 | 4 | 1 | 3 | 7 | 6 | 9 |
| 7 | 1 | 4 | 8 | 6 | 9 | 5 | 3 | 2 |
| 8 | 5 | 2 | 3 | 9 | 6 | 4 | 1 | 7 |
| 3 | 4 | 9 | 7 | 8 | 1 | 6 | 2 | 5 |
| 1 | 7 | 6 | 2 | 5 | 4 | 9 | 8 | 3 |

**Guess 7 or 2; or Start Anywhere**

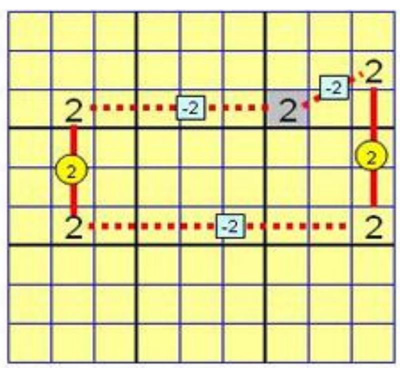
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 | 6 | 9 | 8 | 3 | 2 | 7 | 5 |
| 7 2 | 8 | 9 | 6 | 5 | 2 7 | 1 | 4 | 3 |
| 3 | 2 7 | 5 | 1 7 | 1 2 | 4 | 9 | 6 8 | 6 8 |
| 1 | 3 | 2 | 4 8 | 4 6 | 5 | 7 | 6 8 | 9 |
| 6 7 | 5 | 4 | 7 8 | 9 | 1 6 7 | 3 | 2 | 1 6 8 |
| 9 | 6 7 | 8 | 2 | 3 | 1 6 7 | 4 | 5 | 1 6 |
| 8 | 4 | 1 | 3 | 2 6 | 2 6 | 5 | 9 | 7 |
| 5 | 9 | 7 | 1 4 | 1 4 | 8 | 6 | 3 | 2 |
| 2 6 | 2 6 | 3 | 5 | 7 | 9 | 8 | 1 | 4 |

**A Turbot Fish**: is a single-digit solving technique that uses a loop of odd length. The minimum length is 5, forming a fish-shaped pattern that gave the technique its name. Longer loops are possible, but they are better known as Fishy Cycles.

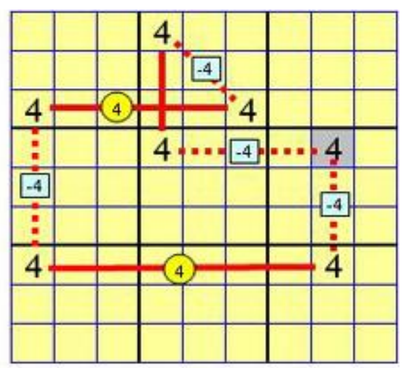
The **orange** cells form 3 rows (4,7,8). There are only two cells in each of these rows with the candidate 2. A perfect True False Scenario. If we assume Cell (8,5)=2, then there is no room for a 2 in row 4. Therefore Cell (8,9)=2.



Turbot fish
x-cycle with discontinuity at
shaded cell = 7



Turbot fish
x-cycle with discontinuity at
shaded cell <> 2



x-cycle with discontinuity at
shaded cell <> 4

**TRUE FALSE Cycle:** The above graphs look more complicated than they really are. In any of the graphs, start with any number as **TRUE** and cycle through, clockwise or counter clockwise, **False, True, False, True, etc.** Eventually, when the chain is completed **it ends with a contradiction**.

For example; look at the grid on the left with the 7s. Let's start in cell (3,1) assuming it is **TRUE.** Then cycle through, clockwise or counter clockwise until you return to cell (3.1). So we have T, F, T, F, T, F. When we return to Cell (3,1) it is **FALSE**. The starting number is both **TRUE & FALSE.**

To resolve the contradiction, the candidate in the grey box, i.e. 7 in cell (8,8) must be eliminated. However, we have not resolved the other candidates. We are left with a chain of 7s. But at least the chain is consistent.
The logic is the same for the middle grid and right grid. The value in the Gray Cell must be eliminated from the Chain.

Varieties of Sudoku: http://www.sudokudragon.com/sudokuvariants.htm

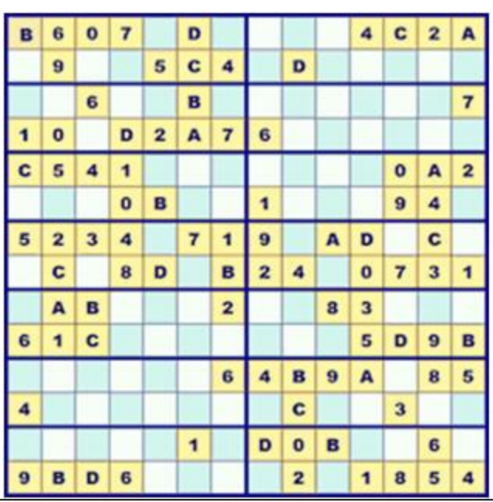
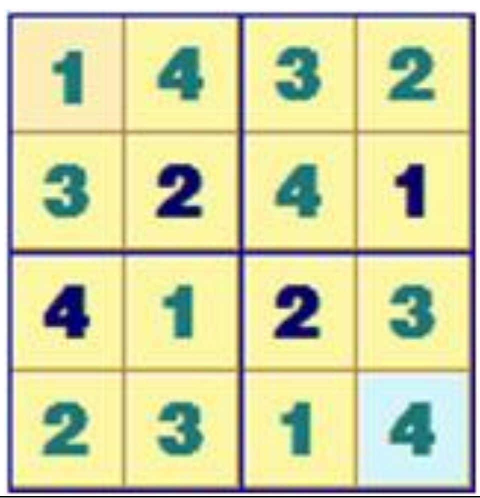
The sudokudragon.com website is very informative and a recommended source of information about Sudoku, particularly variations of the puzzle. Many of the illustrations below have been obtained from their website, hopefully without violating their copyright, if any.

With such a simple rule for Sudoku, you can apply the same idea in lots of different ways. First of all you can change the size of the grid. Using the standard 9x9 grid is only one option. The simpler 4x4 grid is useful for learning the basics of Sudoku. There are only four symbols and four regions to consider, but 4x4 never makes a hard puzzle.

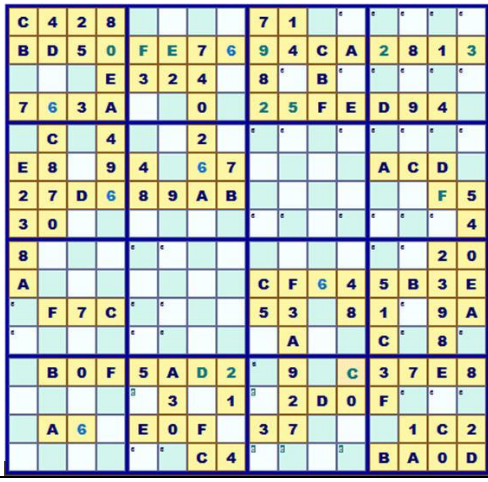
Stepping up the other way 16x16 grid makes a big challenge, because there are 16 cells in a Group and 16 possibilities for each cell. It is not possible to use just digits, so letters or symbols or a combination of numbers and letters and symbols. Same is true when we step up to 25x25 and then 36x36 and so on, but 16x16 with a total of 256 cells to complete is challenging enough. After that, the level of complexity the puzzle has too many possibilities to carry around in the average sized head.

You can also make the regions rectangular rather than square, for example a 2x3 grid (about the most common rectangular size you will find) and the 4x5 monster sized grid.

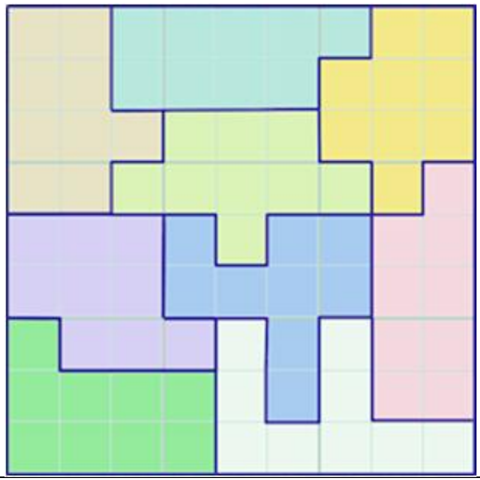
The recommend website describes a number of different forms of Sudoku with example grids. Some newspapers print super-sudoku with overlapping 3x3 puzzles so that one region of the grid is shared by a central 3x3 grid. These are time consuming but rewarding when solved.

| **2x7 sub-grid=14x14 grid** | **4x4 grid** |
|---|---|
|  |  |
| | |

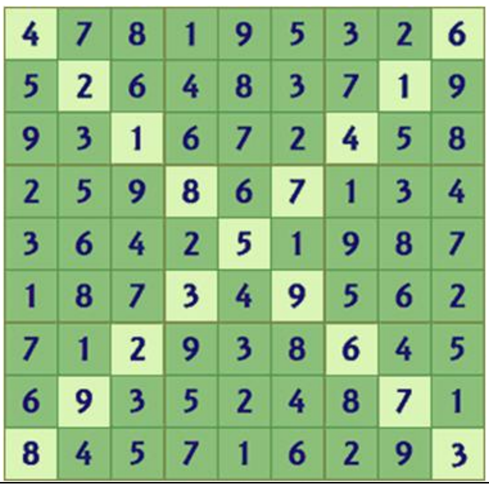
## 16x16 grid below



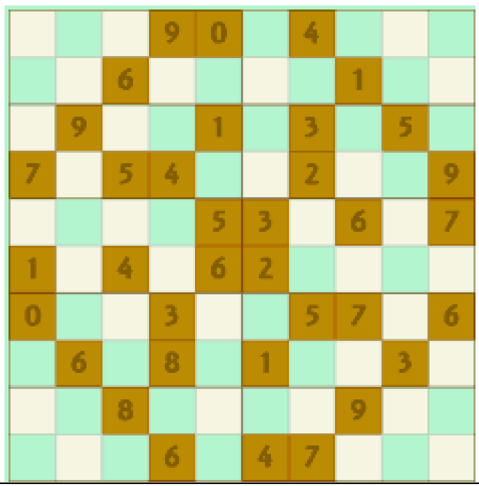

## 9x9 grid=jigsaw below



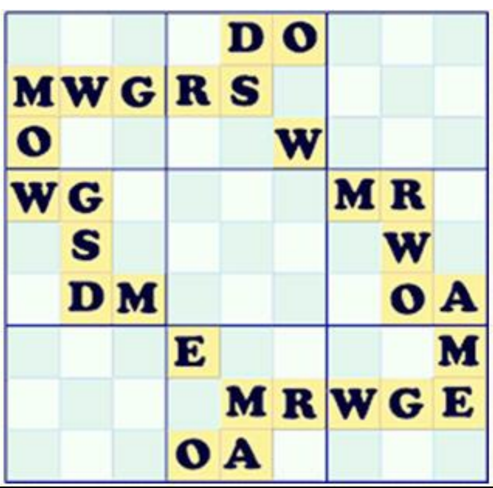
## 9x9 grid plus diagonals



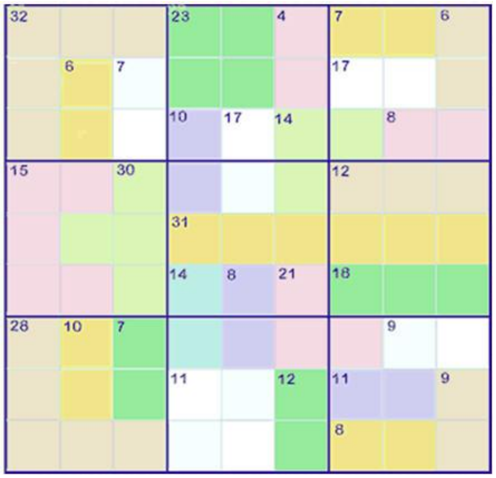
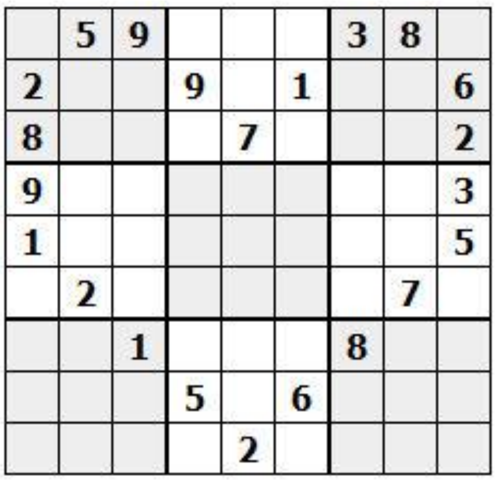
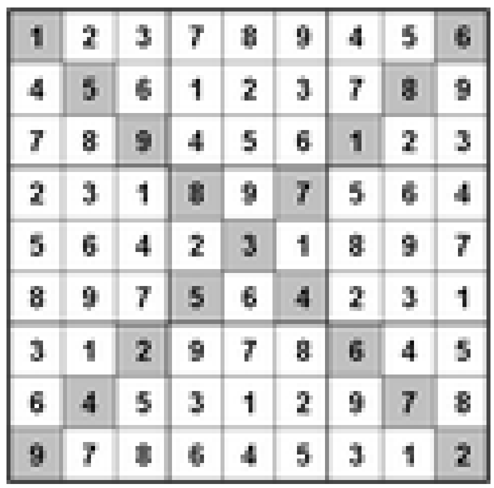
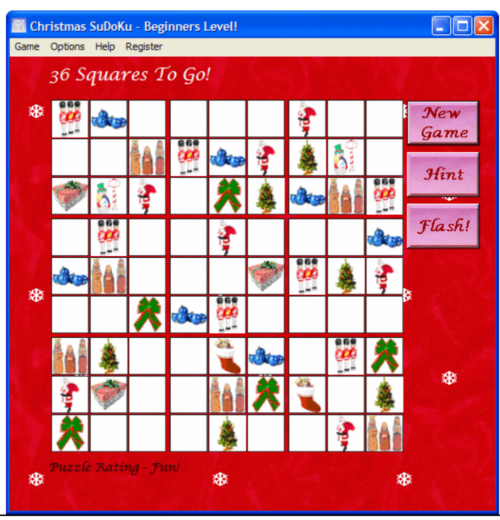
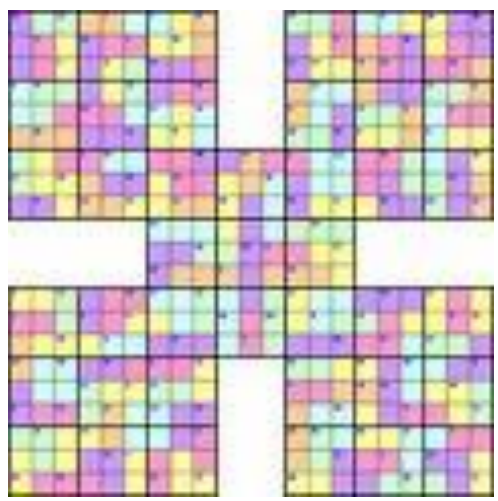
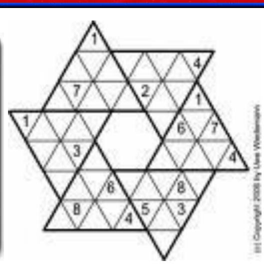
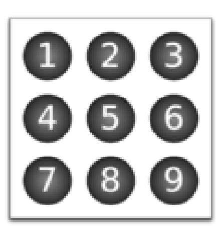
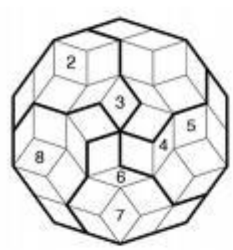

## 2x5 grid=10x10

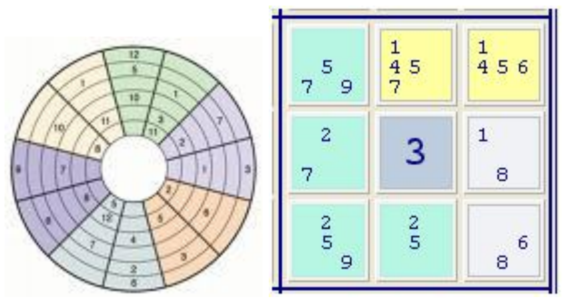


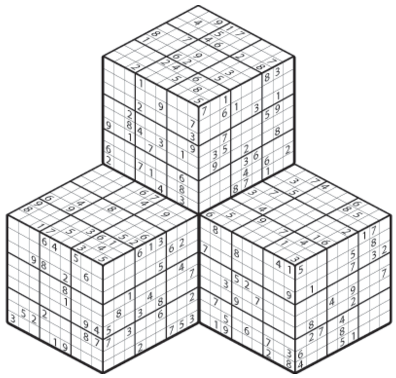

## Wordgame=letters

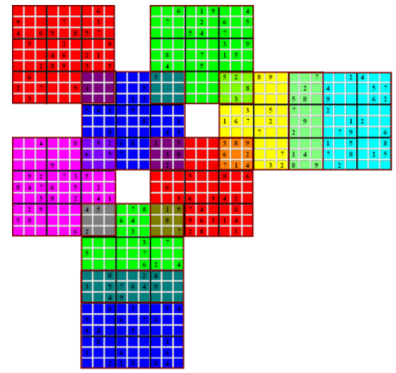



## Wordgame=answer

| Killer Sudoku = Sums | Killer Sudoku answer |
|---|---|
|  |  |
|  |  |
|  |  |
|   |   |





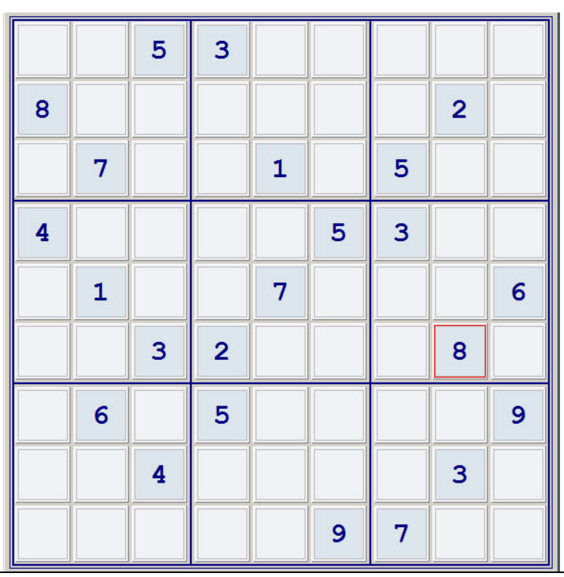


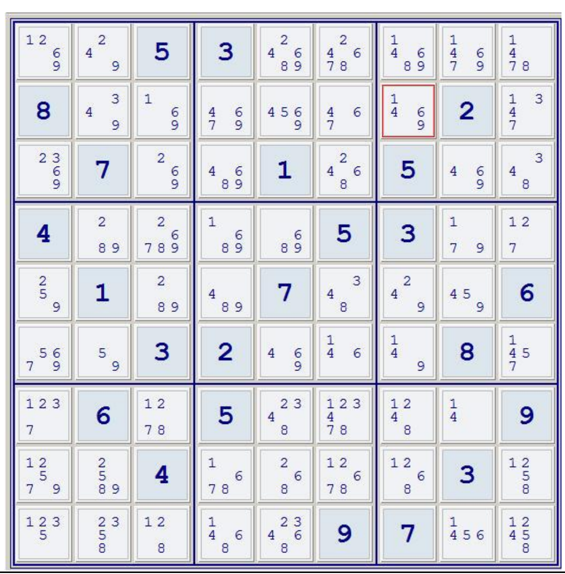
**World's Toughest Sudoku Puzzle:** **Finnish mathematician Art Inkala developed this puzzle which requires billions of computer calculations to solve. Several claimed their computer program solved the puzzle in less than a second. You can read more about it at the foxnews link below.**

Answer is . . . 1 4 5 3 2 7 6 9 8 8 3 9 6 5 4 1 2 7 6 7 2 9 1 8 5 4 3 4 9 6 1 8 5 3 7 2 2 1 8 4 7 3 9 5 6 7 5 3 2 9 6 4 8 1 3 6 7 5 4 2 8 1 9 9 8 4 7 6 1 2 3 5 5 2 1 8 3 9 7 6 4

IMPORT VERSION: 005300000800000020070010500400005300010070006003200080060500009004000030000009700

http://www.foxnews.com/scitech/2010/08/19/crack-worlds-toughest-sudoku/#content





| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 5 8 | 2 | 3 | 6 | 4 5 | 7 | 9 | 4 8 | 1 |
| 5 8 | 6 | 1 | 4 5 | 2 | 9 | 4 8 | 3 | 7 |
| 9 | 4 | 7 | 3 | 1 | 8 | 2 | 6 | 5 |
| 2 | 1 7 | 9 | 8 | 3 4 7 | 1 3 | 4 7 | 5 | 6 |
| 4 | 1 7 | 5 8 | 9 | 6 | 1 5 | 7 8 | 2 | 3 |
| 6 | 3 | 5 8 | 4 5 7 | 4 5 7 | 2 | 1 | 4 8 | 9 |
| 3 | 5 | 2 | 1 | 9 | 4 | 6 | 7 | 8 |
| 7 | 9 | 6 | 2 | 8 | 3 5 | 3 5 | 1 | 4 |
| 1 | 8 | 4 | 5 7 | 3 5 7 | 6 | 3 5 | 9 | 2 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 3 | 9 | 7 | 6 | 5 | 8 | 2 | 4 | 1 |
| 2 | 1 | 8 | 9 | 4 | 7 | 3 | 6 | 5 |
| 5 | 6 | 4 | 2 | 1 | 3 | 7 | 8 | 9 |
| 1 | 8 | 9 | 4 | 3 | 6 | 5 | 2 | 7 |
| 7 | 3 | 6 | 5 | 2 | 9 | 4 | 1 | 8 |
| 4 | 2 | 5 | 8 | 7 | 1 | 9 | 3 | 6 |
| 6 | 5 | 3 | 1 | 9 | 2 | 8 | 7 | 4 |
| 8 | 4 | 2 | 7 | 6 | 5 | 1 | 9 | 3 |
| 9 | 7 | 1 | 3 | 8 | 4 | 6 | 5 | 2 |

**True-False Chain:** If you assume the 4 in cell (2,7) is true, it forces an 8 in Cell (1,8) which forces a 5 in Cell (1,1) which forces a 4 in Cell (1,5). This creates a dilemma because there is no cell for a 4 anywhere in Row 4. So cell (2,7) must be an 8.

**Solution from page 6.**

**The solution immediately above is from the rotated puzzles on page 6. Enjoy.**






Hand held pocket canididate calculator - $15

# Quick Launch! **This explanation is continued from page 4; in a friendlier (?) reading format.**

**1.** **There are two types of players: beginners and advanced players. The major difference separating the two types is accuracy. Advanced players tend to complete a higher percentage of puzzles than beginners.**

**2.** **Beginners should use 2 or 3 simple techniques (i.e. forces) that will enable them to completely solve simple and moderate puzzles. Examples of techniques,[most are described in these pages with graphic examples]: Alleys, Cross Hatching; Group Cross Hatching, Super Hatching, Virtual Hatching, Claims, Visualization, Muddling, Naked & Hidden Singles Doubles, Triples, rare Quads & Quints, The Force, The Complete, Only Choice, Last Remaining Cell, Slicing and Dicing, etc. The principles underlying all of these techniques are very similar; hopefully, once understood, we can forget all the buzzwords. Which is the most important force? Cross hatching seems to be intuitive; the brain seems to know how to cross hatch. Next in importance is probably The Claim; [ also called the Forcing Power of 2s or 3s; also called doublets or circled pairs or triplets in the case of 3s] within a Group [also called an Area]. A Group can be a row, column or box. If it is determined that a candidate, like the candidate 5 as a trivial example, can exist in only two possible squares or cells, then it cannot exist in the remaining 7 cells in the Group. This observation then leads to Virtual Cross Hatching; and then your intuitive brain will take over.**

**3.** **Advanced players should use Chains of all sorts to solve the most difficult problems. Examples include: Swordfish, Squirmbag, Jellyfish, TurbotFish, X-Wing, Y-Wing, XY-Wing, XYZ-Wing, Uniqueness, Gordonians, Franciscans, Eppstein Bi-value Cycles, True False Chains, Colors, Naked & Hidden Quads and Quints with two values (candidates) per cell. The principles behind chains are all similar. Learn the principles and forgot all the buzzwords.**

**There are many types of chains. Two important categories of chains are described here conceptually: TRUE/TRUE: If A is True, then B is True. E.g. if 5 in a cell pair of (5,6) is True, then in a logically connected pair, the 6 in a cell pair of (5,6) must be True. The second example involves three cells, called TRUE/TRUE/FALSE. So let's extend the example to three cells A, B, C that are logically connected. if the 5 in the A pair (5,6) is T and the 6 in the B pair (5,6) is T and the 5 in the C pair (5,9) is F, then this is ACTIONABLE provided at the same time that when the 5 in A pair (5,6) is F and the 6 in the B pair (5,6) is F and the 5 in the C pair (5,9) is False. I.e. the 5 in the C pair of (5,9) is made False by the logical intersection of the A and B cells. An XY-Wing is a good example of the actionable intersection.**

**4.** **The remainder of Sudoku literature is essentially essential gobblygook: a collection of hard to remember patterns, terms and suggested rules that are probably best forgotten. Unfortunately, it is necessary to learn the tools and patterns used by experienced players. This will lead to a more thorough understanding of the simplicity of chains. We covered all the patterns mentioned in paragraph 4 in the previous sections.**

**5.** **What are the biggest pitfalls in Sudoku? Clerical and logical errors that occur using the various methods of identifying potential candidates for each empty cell. Remember that it is important to keep the candidate list up to date. When a cell or square is solved, the potential helpful candidates in other cells must be removed. Remember that Clerical and Logical errors happen all the time when one is momentarily distracted for just a split second; that's all it takes; let's call it a "senior moment." Nothing is more disheartening than discovering an inconsistency in the End Game caused by a clerical error or a logical error. Advanced players have developed the knowledge and skills to make fewer clerical and logical errors. An important skill is intuitive subitizing – see paragraph 10.**

**6.** **To solve the more difficult puzzles, the player has to increasingly rely on one's short term memory, and remember lots of data points and their relationships to each other within Sudoku Units. Remember that each cell is logically connected to 20 other cells forming a Sudoku Unit. The value in a given cell cannot be repeated in any of the other 20 cells in the Sudoku Unit. This is a HUGE piece of information! How does a player capitalize on it? The player has to develop the visual and mental skills to read "empty cells" quickly and accurately. Practice Subitizing. Wikipedia says: "refers to the rapid, accurate, and confident judgments of number performed for small numbers of items."** See the next section on a subitizing exercise. This is just a sample. The player who desires to excel in Sudoku should consult Wikipedia and other internet resources on **how to practice improving subitizing skills**

**7.** **Although there are only 81 cells in a regular Sudoku, there could be as many as 200 to 400 potential values in play. This is lot of values and relationships to remember! Identifying all the potential values in all empty cells is hard, time consuming work and error prone. If one is 99% accurate in a markup, then they will surely make 3 mistakes (on average), either omissions or commissions. It only takes one mistake to riddle the puzzle with inconsistencies in the End Game.**

**8.** **Paragraph 7 begs the question: how does one proceed systematically and efficiently to keep track of all the data points and their relationships? The answer: it requires a disciplined methodology or Markup System. The method cannot be a random walk around the puzzle page. The player must know exactly where one is on the page. The player must know precisely how many of each of the 9 numerals have been solved or listed as potential candidates; see the illustration on page 17 – keeping track of potential candidates. We have discussed various systems in the previous pages, but good players develop their own systems. Wikipedia and the internet are replete with other examples. The player should be able to put the puzzle down and pick up some time later and know where one had left off. Of course, this is all easier said than done; and we all, from time to time, reach a point where a puzzle STALLS; and we have to dig deeper into our bag of tricks; sometimes to little avail. What to do? See paragraph 11 below.**

**9.** **Most easy and moderate puzzles are solved with repetitive cross hatching and claims. If the player has been faithful in identifying doublets (i.e. the power of 2s) and triplets (the power of 3s), a major chunk of the markup of potential candidates may already be completed. The chore of listing all potential candidates in a row, column of box may then be diminimus.**

**On the very difficult puzzles, one may exhaust all their tools too early in the process; and, will be forced to undertake an exhaustive listing of all candidates which is error prone.**

**10.** **Well then, how does one advance to routinely solve the most difficult puzzles. The author believes that one should employ all the easy techniques on a repetitive basis, paying close attention to the documentation of Claims (i.e. both doubletons and tripletons.) The author believes that delaying the detail markup as long as possible is a winning strategy. The author believes that developing subitizing skills is essential to speed and accuracy. Most people can handle 5 or 6 cells or squares from childhood training. The challenge is to extend one's subitizing range to 9 cells. In the following three examples, most of us know what candidates are missing with minimal conscious thought.**

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 5 | | | 3 | | 2 | 1 | | 5 | 3 | | 4 | 1 | | | 2 |

**11.** **The additional skill that distinguishes advanced players from good players is the ability to subitize Sudoku Units, described on page 9. Each cell or square is part of unique Sudoku Unit of 21 cells created by the intersection of the unique row, column and box that contain the cell. The 20 so called buddy cells of a given cell create exclusionary forces on the given cell in two ways: (1) if a buddy cell contains a value; that value cannot exist in the given cell; (2) if the buddy cell is empty, its potential candidate set is still constrained, which in turn limits the potential candidates in the given cell. For example, a buddy cell may in fact be an "only choice" cell. Each buddy cell belongs to its own unique Sudoku Unit. Clearly, the ability in both speed and accuracy to parse each of the three Groups in the Sudoku Unit is dependent on the subitizing skills that need to be developed per the discussion in the previous paragraph 10.**

**12.** **The final chapter: what to do when all else fails.**

**(1) There is always the possibility that the puzzle is flawed and must be discarded.**
**(2) There is always the possibility that a potential or actual candidate was incorrectly entered.**
**(3) There is always the possibility that the puzzle is too hard. See page 47 – the world's toughest puzzle. The author has not been able to find an algorithm to solve the problem in a logical manner. It can be solved by "brute force" using a downloadable program at http://www.madoverlord.com/Projects/Sudoku.t named Sudoku Susser maintained by Robert Woodhead.**

**Here are some suggestions:**

**(1) Put the puzzle down and revisit it later, maybe tomorrow.**
**(2) Start over with clean grid; enter carefully all the potential candidates at the get go. (use Simple Sudoku as an aid)**
**(3) Play the puzzle in the computerized program Simple Sudoku, by Angus Johnston, which can be downloaded from http://www.www.angusj.com/sudoku. It will accurately fill in all potential candidates, and provide a great learning experience. Chances are it will uncover the move that breaks the logjam.**
**(4) If Simple Sudoku stalls, then go to the website: http://www.scanraid.com/sudoku.htm which is maintained by Andrew Stuart. This site identifies step by step all the analyses techniques from the most simple crosshatch to complex chains. It is powerful and will provide a very rich learning experience for advanced players.**
**(5) If Andrew Stuart's site fails, the last resort would be http://www.madoverlord.com/Projects/Sudoku.t maintained by Robert Woodhead. The program can be downloaded for the PC, MAC and Linux. It was able to solve the World's Toughest Problem by sheer brute force.**

# Practice Subitizing to Improve Accuracy/Speed

Glance at a row, column or box and estimate (guess) the missing candidate(s). With a lot of practice you will instinctively know which candidates are missing. Practice on rows, columns and boxes. You will eventually surprise yourself, and it will dramatically improve your ability to see hidden patterns.

## Make up your own patterns and practice on smaller boxes.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 8 | | 4 | 7 | 6 | 5 | 3 | 2 | |
| 7 | 3 | 9 | | 2 | 1 | 8 | | 5 |
| 6 | 5 | | | 3 | 9 | 1 | 4 | 7 |
| | 2 | | 6 | 9 | 4 | 5 | 3 | 8 |
| 9 | | 6 | 5 | 7 | | 2 | 1 | 4 |
| 5 | 4 | 3 | 2 | 1 | 8 | | | 6 |
| 2 | 7 | 1 | 9 | 8 | | | 5 | 3 |
| | 6 | 8 | 3 | 5 | 2 | 7 | 9 | |
| 3 | 9 | 5 | 1 | | 7 | 6 | 8 | 2 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | | 9 | 5 | 6 | 2 | | 3 | 4 |
| 3 | | 7 | | 9 | 4 | 5 | 8 | 6 |
| | 4 | 5 | 3 | 8 | | 9 | 2 | 1 |
| 5 | 7 | | 2 | 4 | 1 | 6 | 9 | |
| | 3 | 1 | | 7 | 9 | 2 | 5 | 8 |
| 9 | 6 | | 8 | 5 | 3 | 1 | | 7 |
| 8 | 1 | 6 | 4 | 2 | 5 | 3 | | |
| 2 | 9 | 3 | 7 | | 8 | | 6 | 5 |
| 7 | 5 | 4 | 9 | | | 8 | 1 | 2 |

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|---|---|---|---|---|---|---|---|---|
| | 7 | 8 | 5 | 2 | 3 | 4 | 1 | 9 |
| 5 | | 4 | 1 | 9 | 8 | 3 | 7 | 6 |
| 3 | 1 | | 7 | 6 | 4 | 2 | 5 | 8 |
| 1 | 4 | 3 | | 5 | 2 | 9 | 8 | 7 |
| 2 | 9 | 5 | 8 | | 1 | 6 | 4 | 3 |
| 7 | 8 | 6 | 3 | 4 | | 5 | 2 | 1 |
| 9 | 6 | 2 | 4 | 1 | 7 | | 3 | 5 |
| 8 | 5 | 1 | 2 | 3 | 6 | 7 | | 4 |
| 4 | 3 | 7 | 9 | 8 | 5 | 1 | 6 | |

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| 9 | 6 | 3 | 1 | 7 | 4 | 2 | 5 | |
| 1 | 7 | 8 | 3 | 2 | 5 | 6 | | 9 |
| 2 | 5 | 4 | 6 | 8 | 9 | | 3 | 1 |
| 8 | 2 | 1 | 4 | 3 | | 5 | 9 | 6 |
| 4 | 9 | 6 | 8 | | 2 | 3 | 1 | 7 |
| 7 | 3 | 5 | | 6 | 1 | 8 | 2 | 4 |
| 5 | 8 | | 7 | 1 | 3 | 4 | 6 | 2 |
| 3 | | 7 | 2 | 4 | 6 | 9 | 8 | 5 |
| | 4 | 2 | 5 | 9 | 8 | 1 | 7 | 3 |